D 31897 (Pages : 3) Name

Reg. No.....

SECOND SEMESTER B.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION DECEMBER 2012

(CCSS)

MM 2C 02—MATHEMATICS

Time : Three Hours Maximum : 30 Weightage

I. Objective Type Questions : (Answer *all* questions, weight 12 × ¼ = 3)

1. Integrate coth 5x.

2. Investigate the convergence of $\int \frac{dx}{x^2}$

3. Find $\lim \frac{2^n}{5n}$

4. Define Absolute convergence.

5. Find a formula for the nth term of the sequence 1, —1, 1, —1, 1, ...

6. State the Sandwich theorem for sequences.

7. Examine the convergence of $\sum_{n=1}^{\infty} \frac{2n}{n}$

8. Graph the set of points whose polar co-ordinates satisfy the conditions —3 r 2 and $0 = \frac{}{4}$

9. Find the polar equation of the hyperbola with eccentricity $\frac{3}{2}$ directrix x = 2.

10. Define the continuity at (x_0, y_0) of the function $f(x, y)$.

11. Find $\frac{\partial^2 w}{\partial x \partial y}$ if $w = xy + \frac{}{y^2 + 2}$

12. Find f_x if $f(x, y) = \frac{}{y + \cos x}$.

Turn over

II. Short Answer Type Questions : (Answer all *nine* questions, weight 9 x 1 = 9)

13. Evaluate $\int \frac{dx}{1+x}$

14. Examine the convergence of $\sum_{n=1} \frac{(-1)^n 5}{4^n}$

15. Does the series $5 + \frac{2}{3} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{k!}$ + ... converge ?

16. Examine the convergence of $\sum_{n=1} \frac{4^n n!n!}{(2n)!}$

17. For what values of x does the power series $\sum_{n=0} n!x^n$ converges.

18. Find the distance from one focus of the ellipse with semimajor axis 39.44 AU and eccentricity 0.25, to the associated directrix.

19. Find $\left(\frac{\partial w}{\partial x}\right)_{y,z}$ if $w = x^2 y - z + \sin t$ and $x + y = t$.

20. Find $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$

21. If Resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from the equation $\frac{1}{R} = \frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{3}$. Find the value of $\frac{\partial R}{\partial R_2}$ where $R_1 = 30$, $R_2 = 45$, $R_3 = 90$ ohms.

III. Short Essay Questions : (Answer any *five* questions, weight 5 x 2 = 10)

22. Evaluate $\int_0 \frac{2dx}{3 + 4x^2}$

23. **Sum the series** $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^n - 1}$.

24. Prove that $\tanh^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} +$

25. Graph the **cardioid** $r = 1 - \cos\theta$.

26. The plane x = 1 intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1, 2, 5).

27. Find $\frac{dw}{dt}$. $w = xy + z$, $x = \cos t$, $y = \sin t$, $z = t$.

28. If $z = x + f(u)$ where $u = xy$ show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x$.

IV. Essay Questions : (Answer any *two* questions, weight 2 x 4 = 8)

29. Does the series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots + \frac{1}{n^2} + \ldots$ converge ?

30. Multiply the geometric series $\sum_{n=0} x^n = 1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x}$, for $|x| < 1$, by itself to get a power series for $\frac{1}{(1-x)^2}$.

31. Find the linearization L (x, y) of $f(x, y) = 1 + y + x\cos y$ at P_0 (0, 0) and find an upper bound for |E| of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle R $|x| \le$ $|y| \le$