D 1824 (Pages : 3) Name.......

Reg. No..............................

SECOND YEAR B.Sc. DEGREE EXAMINATION
SEPTEMBER/OCTOBER 2009

Part III—Mathematics–(Subsidiary)

aper II—ALGEBRA, COMPLEX NUMBERS, MATRICES VECTORS AND DIFFERENTIAL EQUATIONS

(2001 Admissions)

Time : Three Hours Maximum : 95 Marks

Maximum marks that can be obtained from Unit I is 35, *Unit* II *is* 25 *and Unit* III *is* 35.

Unit I (Algebra, Complex Numbers, Matrices)

Maximum marks that can be obtained from this unit is 35

1. **..n the infinite series** $\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+\cdots$ **(5 marks)**

2. bind the sum **to infinity of** $\frac{1^3}{1!}+\frac{2^3}{2!}+\frac{3^3}{3!}+\cdots$ **(6 marks)**

3. Prove that $\log_e x = \frac{x-1}{x+1}+\frac{1}{2}\frac{(x-1)^2}{(x+1)^2}+\frac{1}{3}\frac{(x-1)^3}{(x+1)^3}+\cdots$ **(6 marks)**

4. **If $\sin(A+iB)=x+iy$, show that**
$\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$ **and** $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$ **(6 marks)**

5. **Express sin 5θ in terms of sin θ** **(5 marks)**

6. **Express tan 4θ in terms of tan θ** **(5 marks)**

7. **Prove that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$** **(6 marks)**

8. **By reducing into normal form , find the rank of the matrix A =** $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 2 & 3 & 9 & -10 \\ 16 & 4 & 10 & 15 \end{bmatrix}$ **(5 marks)**

9. **Find the values of k, for which the equations**

$$x+y+z=1,$$
$$x+2y+3z=k \quad \text{and}$$
$$x+5y+9z=k^2$$

have a solution. For these values of k, find the solutions also. **(10 marks)**

Turn over

10. Find the **eigen** values and **eigen** vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ **(8 marks)**

11. If A is an **eigen** value of an orthogonal matrix , show that $\frac{1}{\lambda}$ is also its **eigen** value. **(4 marks)**

12. Using **Cayley-Hamilton** theorem, find A where A = $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ **(4 marks)**

Unit II (Vectors)

Maximum marks that can be obtained from this unit is 25

13. Prove that $[\vec{a}, \vec{b} + \vec{c}, \vec{a} +$ $= 0$ **(5 marks)**

14. Prove that $[\vec{a}$ x b, $\vec{b} \times \vec{c}, \vec{c} \times$ $= [\vec{a}, \vec{b}, \vec{c}]^2$ **(5 marks)**

15. Find the value of A such that the vectors $\vec{i} + 27$— $3\vec{k}$, 27+ $3\vec{i} + \lambda\vec{j} + \vec{k}$ are coplanar. **(4 marks)**

16. Find the angle between the tangents to the curve $x = 5t^2, y = t, z = 3 - t^3$ at the points $t = \pm 1$.. **(4 marks)**

17. The acceleration of a particle at a time t is given by $f =$ ——— $3t\ \vec{k}$ Find the velocity vector v, given that $\vec{v} = 2$i $+ j$ when $t = 0$. **(5 marks)**

18. Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,2, —1) **(4 marks)**

19. Find the value of A , **if** $P = (2x - 5y)\vec{i} + (x + \lambda y)\vec{j} + (3x - z)\vec{k}$ is solenoidal. **(4 marks)**

20. Show that $P = (x^2 -$ $(y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is **irrotational** and hence find its scalar potential. **(8 marks)**

21. Show that **div grad** $\left(\frac{1}{}\right) = 0$. **(6 marks)**

22. Using the line integral , **compute the** work done by the force $P = (2y + 3)\vec{i} + xz\vec{j} + (yz \quad x)\vec{k}$ **when it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the path** $x = 2t^2$, y = t, z = **(5 marks)**

Unit III

Maximum marks that can be obtained from this unit is 35

23. **Solve** $(y^2 + xy\ )dx + (x^2 - yx\ )dy = 0$ **(4 marks)**

24. **Solve** $x\frac{d}{dx} + y = x\ y$ **(6 marks)**

25. **Solve** $(D\ -4D+13)y = e\ ^{x}\cos 3x$ **(6 marks)**

26. **Solve** $(D\ -2D\ +D\ )y = x^2 + ex$ **(6 marks)**

27. **Find** $L\,(e^{-t}\cos\ t)$ **(4 marks)**

28. **Using convolution theorem find the inverse transform of** $(s2\quad a^2)^2$ **(6 marks)**

29. **Using Laplace transform solve y" +2y = 4, given that y(0) = 2, $y\ (0) = 3$** **(8 marks)**

30. **Find the Fourier series for the function defined by**
$f(x) = x^2 + x\ in\ -\pi < x < 7r$ **and** $f(x + 2\pi) = f(x)$ **(7 marks)**

31. **Obtain** the half-range cosine series for the function

$$f(x) = \begin{cases} \cos x & , 0 < x < \frac{7r}{2} \\ 0 & \frac{\pi}{2} < x < 7r \end{cases}$$

(5 marks)

32. Using Taylor's series method , find y(0.1) and y(0.2)
given that $\frac{d}{dx} = x + y$ and y(0) = 1 **(8 marks)**

33. Using **Euler's** method , compute y(0.2) in steps of 0.1 if $\frac{dy}{dx} = x - y$ **and y(0) = 1** **(10 marks)**