D 8816 (Pages : 3) Name..............................

Reg. No..............................

SECOND YEAR B.Sc. DEGREE EXAMINATION SEPTEMBER/OCTOBER 2010

Part III—Mathematics (Subsidiary)

Paper II—ALGEBRA, COMPLEX NUMBERS, MATRICES, VECTORS AND DIFFERENTIAL EQUATIONS

(2001 admissions)

Time : Three Hours Maximum : 95 Marks

Unit I (Algebra, Complex Numbers, Matrices)

Maximum marks that can be obtained from this Unit is 35.

1. Simplify : $\dfrac{(\cos 50 - i\sin 50)^2 (\cos 7\theta - i\sin 7\theta)}{(\cos 40 - i\sin 40)^9 (\cos 0 + i\sin 0)^5}$ (5 marks)

2. Prove that $\dfrac{\sin 70}{\sin 0} = 8\cos 20 + 4\cos^{t} 20 - 4\cos 20 - 1$. (5 marks)

3. Prove that $32\sin 0 \cos^{t} 0 = \cos 60 - 2\cos 40 - \cos 20 + 2$. (5 marks)

4. Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. (7 marks)

5. Prove that $\sinh z = \log\left[z + \sqrt{z} + 1\right]$. (5 marks)

6. If $\dfrac{(1-3x)^{1/} + (1-43}{-5x}$ is approximately equal to $a + bx$ for all small values of x. Find a and b. (5 marks)

7. Find the coefficient of x^n in $\dfrac{1 + 2x - 3x^2}{e^x}$ (5 marks)

8. Show that $\left(1 + \dfrac{1}{n}\right)^{\frac{1}{2}} = e\left(1 + \dfrac{1}{12n^2}\right)$ nearly when n is large. (6 marks)

Turn over

9. Reduce the following matrice to normal form and hence find its rank $\begin{vmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{vmatrix}$

(6 marks)

10. Determine the values of a and b for which the system

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

$2x + 5y + az \quad b$

has (i) no solution ; (ii) unique solution ; and (iii) infinite number of solution.

(7 marks)

11. Find the eigen values and eigen vectors of $\begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$. (7 marks)

12. Find A^{-1} if $A = \begin{vmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{vmatrix}$ using Cayley-Hamilton theorem. (6 marks)

Unit II (Vectors)

Maximum from this unit is 25.

13. Prove that $\left[\bar{a} + b, \qquad + \quad \right| = 2\left[\bar{a}, \bar{b}\ \bar{c}\right]$. (5 marks)

14. If a and b lie in a plane normal to the plane containing $\bar{c}$ and a. Show that $(\bar{a} \times 6) \cdot (\bar{c} \times \bar{d}) = 0$.

(5 marks)

15. Given $a = 2i + 2k$, $\bar{b} = 4i + 4j - 2k$, $= i - 2j + 3k$, $d = 2i - j + 5k$. Find $(a + \quad x \left(\bar{b} \times d\right)$. (5 marks)

16. If a and b are constant vectors prove that $V\left(\bar{r}\ a \quad = a \times b\right.$. (6 marks)

17. If a is a constant vector. Prove that :

(i) r x a is solenoidal.

(ii) Curl $(\bar{r} \times a) = a$.

(iii) Grad $(\bar{a} \cdot \bar{r}) = a$.

(8 marks)

18. Show that r^n r is an irrotational vector for any value of n but is solenoidal only if n = -3.

(6 marks)

19. Prove that $\nabla^{\hat{}} \left(r'' \bar{r}\right) = n\,(n+3)\,r \quad \hat{} \;$ r.

(7 marks)

20. If f $= xzi + yzj + z^{\hat{}}k$. Evaluate along the rectilinear path from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0) and then to (1, 1, 1).

(6 marks)

21. Find the angle between the surface $x^2 + y^2 - z = 3$ and $x^2 + y^2 + z^2 = 9$ at a point (2, -1, 2).

(7 marks)

Unit III (Differential Equations)

Maximum from this unit is 35.

22. Solve $\left(xy^2 \quad x^{3}y^{2}\right) dy + \left(1 + y^{\hat{}}\right) dx = 0$.

(6 marks)

23. Solve $\left(x + 2y - 2\right) dy + \left(2x + y + 2\right) dx = 0$.

(6 marks)

24. Solve $\dfrac{dy}{dx} + \dfrac{x}{1+x^2}Y = \dfrac{1}{2x^{\hat{}}\,(1+x^{\hat{}})}$.

(6 marks)

25. Solve $\dfrac{dy}{dx} + y \tan x = y^3 \sec x$.

(7 marks)

26. Solve $(D^2 + 2D - 3)\, y = e^x\, coax$.

(6 marks)

27. Find the inverse transform of $\cot^{-1}(s + 1)$.

(6 marks)

28. Determine the Fourier expansion of x sin x in

(9 marks)

29. Show that in the range 0 n x (n x) $\dfrac{R^2}{=6}$ $\dfrac{\cos 2x}{1_2}$ $\dfrac{\cos 4x}{22}$ $\dfrac{\cos 6x}{32}$ + . . .

(9 marks)

30. Solve by Taylor series method $\dfrac{dy}{dx} = xy^{\hat{}} + 1$, given y(0) =1 at $x = 0.2$.

(7 marks)

31. Using modified Euler's method solve $\dfrac{dy}{dx} = y + x2$, y(0) =1 to find y (0.2) correct to 3 decimals.

(7 marks)