

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2012

Mathematics—Complementary Course

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 30 Weightage

I. Answer *all* questions. Each question of weightage1 When is $M(x, y) dx + N(x, y) dy$ an exact differential equation ?

2 What is the Bernoulli equation ?

3 Solve : $y'' =$

4 Define rank of a non-zero matrix 'A'.

5 Are the matrices $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$ equivalent ?6 What are the characteristic roots of $A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 9 & -5 & 3 \end{bmatrix}$ 7 What is the divergence of $a = [3xz, 2xy, -yz]$?8 What is the volume of a parallel piped with edge vectors \vec{a}, \vec{b} and ?

9 State Laplace's equation.

10 If a surface S is given by $g(x, y, z) = 0$, what is the unit normal vector to S ?11 Give the parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$

12 Give an example of a non-orientable surface.

(12 x $\frac{1}{4}$ = 3 weightage)II. Answer *all* questions. Each question of weightage 1.13 Solve : $(1 - \frac{1}{x}) dx = 1 + x.e^x, y(0) = 1$ 14 Find an integrating factor for $(2 \cos y + 4x) dx = x \sin y dy$.15 Find the rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 7 & 8 \end{bmatrix}$

Turn over

16 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, use **Cayley** Hamilton theorem to find A^4 .

17 If $\vec{a} = [1, 1, 0]$, $\vec{b} = [3, 2, 1]$ and $\vec{c} = [1, 0, 2]$, find the angle between \vec{a} and \vec{b} .

18 Find the tangential and normal accelerations of $\vec{r}(t) = 5t^2 \hat{k}$.

19 Prove that $\text{curl}(\text{grad } f) = \vec{0}$.

20 Check for path independence : $3z^2 dx + 6xz dz$.

21 Use Green's theorem to find the area enclosed by the circle $x^2 + y^2 = 9$.

(9 x 1 = 9 weightage)

III. Answer any *five* questions from seven. Each question of **weightage** 2.

22 Solve : $(2x - 4y + 5)y' + (x - 2y + 3) = 0$.

23 Find the rank by reducing to normal form.:

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

24 Find the **eigenvalues** of $A = \begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

25 Find the directional derivative of $f = xyz$ along $[1, -2, 2]$ at $(-1, 1, 3)$.

26 Test for exactness and hence evaluate :

$$\int_{(0,0,0)}^{(a,b,c)} 2xy \, dx + 2x^2 y \, dy + dz$$

27 Find the length of the catenary $\vec{r}(t) = t \hat{i} + \cosh t \hat{j}$ from $t = 0$ to $t = 1$.

28 Evaluate $\oint_S \vec{F} \cdot d\vec{A}$ using the Divergence theorem, where :

$$\vec{F} = [x^2, 0, z^2] \text{ and } S \text{ is the box } |x| \leq 1, |y| \leq 3, |z| \leq 2.$$

(5 x 2 = 10 **weightage**)

IV. Answer any *two* questions. Each question of weightage 4.

29 Find the Orthogonal trajectories of $y = c \cdot e^x$.

30 Verify Cayley Hamilton theorem for :

$$A = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

31 Verify Stokes theorem for $\vec{F} = x^2, 5x, 0$ and S is the square $0 \leq x \leq 1, 0 \leq y \leq 1, z = 1$.

(2 x 4 = 8 weightage)