

**THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2015**

(CUCBCSS—UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

**Part A (Objective Type Questions)***Answer all twelve questions.*

1. Write the general form of Bernoulli's differential equation.
2. Find the solution of the differential equation  $y' =$
3. What is the order of the differential equation  $y'' - (y')^3 + 4 = 0$ ?
4. State Cayley Hamilton theorem.
5. What is the rank of a  $(n \times n)$  non-singular matrix ?
6. Write the normal form of the matrix :  $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
7. Write the parametric equation of the curve  $\frac{x^2}{4} + \frac{y^2}{3} = 1$ .
8. Define Irrotational vector.
9. Find curl  $\mathbf{v}$ , where  $\mathbf{v} = [2y, 5x, 0]$ .
10. Find the tangent to the curve  $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j}$  at  $(1, 1, 0)$ .
11. Define scalar potential of a vector.
12. State Gauss's divergence theorem.

(12 x 1 = 12 marks)

**Part B (Short Answer Type Questions)***Answer any nine questions.*

13. Find the orthogonal trajectories of the family of curves  $y = ce^x$ .
14. Write the condition for the differential equation  $Mdx + Ndy = 0$  become exact. What is the form of its solution ?

**Turn over**

15. Find the integrating factor of the linear differential equation  $y' - y = e^{2x}$
16. Find characteristic roots of the matrix :  $\begin{vmatrix} -1 & 2 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{vmatrix}$
17. Write the elementary transformations in a matrix.
18. Find the component of vector  $\mathbf{a} = [4, 2, 0]$  in the direction of  $\mathbf{b} = [1, -1, 2]$ .
19. Find the directional derivative of  $xyz$  at the point  $\mathbf{P}(-1, 1, 3)$  in the direction of  $\mathbf{j} - 2\mathbf{j} + 2\mathbf{k}$ .
20. Find the unit normal to the level surface  $z^2 = 4(x^2 + y^2)$  at the point  $P(1, 0, 2)$ .
21. Find  $\text{div } \mathbf{v}$ , where  $\mathbf{v} = xyz\mathbf{i} + 3zx\mathbf{j} + zk$ .
22. Define Jacobian.
23. Find value of  $X$  if  $\mathbf{a} = [4, 2, X]$  and  $\mathbf{b} = [2, -3, 1]$  are orthogonal.
24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 x 2 = 18 marks)

**Part C (Short Essay Type Questions)***Answer any six questions.*

25. Solve the initial value problem  $y' + y \tan x = \sin 2x$ ,  $y(0) = 1$ .
26. Solve  $xy' = y + 3x^4 \cos^t(x)$
27. Find the eigenvalues and eigenvector corresponding to any one eigenvalue of the matrix :  $\mathbf{A} = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{pmatrix}$
28. Use Cayley Hamilton theorem to find  $A^{-1}$  and  $A^4$ , where  $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$
29. Find the tangential and normal components of acceleration of an object moving along the curve  $\mathbf{r}(t) = e^t \mathbf{i} + e^{-t} \mathbf{j}$ .

30. Find tangent to the ellipse  $\frac{1}{4}(x^2 + y^2)$  at the point  $P(1, 1)$ .
31. Find the area of the cardioid  $r = a(1 - \cos \theta)$ ,  $0 \leq \theta \leq 2\pi$ .
32. Evaluate the double integral  $\iint_R y^2 \, dx \, dy$  where R is the region bounded by the unit circle in the first quadrant.
33. Verify Green's theorem in the plane for the vector  $F = (y^2 - 7y)\mathbf{i} + (2xy + 2x)\mathbf{j}$  and the region bounded by the  $x^2 + y^2 = 1$ .

(6 x 5 = 30 marks)

**Part D (Essay Type Questions)***Answer any two questions.*

34. Test for consistency and solve the following system of equation.

$$\begin{array}{ll}
 x + y + z + 3 = 0 & 3x + 26y + 2z = 9 \\
 \text{(a) } 3x + y - 2z + 2 = 0 & \text{(b) } 5x + 3y + 7z = 4 \\
 2x + 4y + 7z - 13 = 0. & 7x + 2y + 10z = 5.
 \end{array}$$

35. (a) Solve the differential equation :

$$2 \sin(y^2) \, dx + xy \cos(y^2) \, dy = 0, y(2) = \frac{\pi}{2}.$$

- (b) Prove that  $\text{Curl}(\text{grad} f) = 0$ .

36. Verify Stokes's theorem for  $F = [y, z, x]$  over the surface of the paraboloid  $z = 1 - (x^2 + y^2)$ ,  $z \geq 0$ .

(2 x 10 = 20 marks)