

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2011

(CCSS)

Mathematics—Complementary

MM 3C 03—MATHEMATICS

Time : Three Hours

Maximum Weightage : 30

I. Objective Type Questions.*Answer all questions.*

1. The degree of the differential equation $(y')^2 + y = x^3 - 4$ is _____
2. The variable separable form of the differential equation $9yy' + 4x = 0$ is _____
3. The necessary and sufficient condition for the differential equation $M(x, y) dx + N(x, y) dy = 0$ is exact is that _____
4. The order of the largest square sub matrix whose determinant is not zero is called _____
5. $|A - \lambda I| = 0$ is called the _____ of A. _____
6. If λ is a characteristic root of a non-singular matrix A then $1/\lambda$ is the characteristic root of _____
7. If the inner product of two vectors is zero then the two vectors are _____
8. The directional derivative of $g, D_b f$, is maximum when \mathbf{b} has the direction of _____
9. When the curl of a vector point function \mathbf{f} is a zero vector then \mathbf{f} is called _____
10. $\text{curl}(\nabla f) =$ _____
11. The line integral $\int \mathbf{F}(r) \cdot d\mathbf{r}$ is path independent if and only if \mathbf{F} is _____ of some function f
12. A domain is called _____ if every closed curve can be continuously shrunk to any point D without leaving D.

$$(12 \times \frac{1}{4} = 3)$$

II. Short Answer Type Questions.*Answer all questions.*

13. Solve the initial value problem $y' = -y/x$; $y(1) = 1$.
14. Find the integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = \frac{1}{3}$
15. Find the general solution of the differential equation $yy' = 4$.

Turn over

16. Obtain the row equivalent canonical matrix C of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$

17. Find the augmented matrix of the following system of equations :

$$x + 2y - 3z = 5$$

$$3x - y = 3$$

$$4x + 3y - z = 2$$

18. Find the eigenvalues of $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$

19. Find a unit vector perpendicular to the vectors $[2, 3, 4]$ and $[-2, 1, 1]$.

20. Find the div $[3x^2, 5xy, xyz]$ at the point $(1, 2, 3)$.

21. Find the directional derivative of $(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point P $(2, 1, 3)$ in the direction of the vector $i - 2k$.

(9 x 1=9)

III. Short Essay or Paragraph Questions.

Answer any **five** questions.

22. Solve the following exact equations $(x^3 + 3xy) dx + (3x^2y + y^2) dy = 0$.

23. Solve the Bernoulli's equation : $y' - Ay = -By^n$ **A, B** positive.

24. Reduce the matrix A to its normal form.

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

25. Prove that the following system of equation is inconsistent.

$$x + y + z + 3 = 0$$

$$3x + y - 2z + 2 = 0$$

$$2x + 4y - 7z - 7 = 0.$$

26. Determine the eigenvectors of the matrix $\begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$

27. Find the work done in moving a particle in the force field $F = 3x^2 \mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$ along the straight line from (0, 0, 0) to (2, 1, 3).

28. Evaluate $\int (z\mathbf{i} + x\mathbf{j} + y\mathbf{k}) \cdot \mathbf{n} \, dS$ where S is the quadrant of the circle $x^2 + y^2 = 1$ between the positive parts of the axes.

(5 x 2 = 10)

IV. Essay Questions.

Answer any two questions.

29. Verify the Cayley-Hamilton theorem and compute the inverse of the matrix

$$A = \begin{vmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{vmatrix}$$

30. Solve: $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y - 14z = 0$.
31. Verify Stoke's theorem for $F = (x^2 + y^2) \mathbf{i} + 2xy\mathbf{j}$ taken round the rectangle bounded by the lines $x = a$, $x = -a$, $y = 0$, $y = b$.

(2 x 4 = 8)