

**FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2016**

(CUCBCSS UG)

Complementary Course

MAT 4C 04—MATHEMATICS

Maximum : 80 Marks

Time : Three Hours

**Part A***Objective type.**Answer all **twelve** questions.*

1. What do you mean by a non linear differential equation ?
2. Write the solution for an homogeneous differential equation with  $2 \pm 3i$  as a double root for its auxiliary equation.  
State the Existence and Uniqueness theorem for initial value problem.
3. What is the Shifting property of Laplace transform ?
5. Find  $L \left( \frac{1}{(s+1)^2} \right)$ .
6. What is  $L(t^{\dots})$  ?
7. How do you define a **sawtooth** wave function ?
8. Define and give an example of an odd function.
9. Write the **Euler's** formula for a **fourier** series of a periodic function.
10. Write the formula for Runge **Kutta** method.
11. Does the **inital** value problem  $xy' = 4y$ ,  $y(0) = 1$  has solution ? Give reason.
12. Give a formula for an error for Simpson's rule. (12 x 1 = 12 marks)

**Part B***Short answer type.**Answer any **nine** questions.*

13. Find the particular integral for  $y' + 4y = 8x^2$ .
14. Find a basis for the solution of the differential equation  $y'' - y = 0$ .
15. Find  $W[$

**Turn over**

16. If  $L^{-1}(f(s)) = F(t)$  then show that  $L^{-1}(f(s-a)) = F(t)$ .
17. Show that the Laplace transform is a linear operation.
18. Find  $L(\sin t)$ .
19. Show that the function  $f(x) = \text{constant}$  is a periodic function of period  $p$  for every positive  $p$ .
20. Find the Fourier series of  $f(x) = \frac{1}{2} - \frac{x}{\pi}$ .
21. Show that  $u = \cos 4t \sin 2x$  is a solution of the wave equation.
22. Apply Picard's iteration upto 4 steps to solve  $y' = y$  and  $y(0) = 1$ .
23. Show that the initial value problem  $y' = y, y(0) = 0$  does not have a unique solution.
24. What do you mean by Lipschitz condition?

### Part C

(9 x 2 = 18 marks)

Short essay.

Answer any **six** questions.

25. Solve  $x y'' + 7xy' + 13y = 0$ .
26. Verify  $y' = 2x^2 - 6x + 7$  is a solution for  $y'' + 3y' + 2y = 4x^2$  and find a general solution.
27. Find the Laplace transform of  $f(t) = \begin{cases} 0, & t < 4 \\ 5, & t > 4 \end{cases}$ .
28. Find  $\frac{3s^2 + 16}{s^2 - s - 6}$ .
29. Solve  $u = 2xyu$ .
30. Find the cosine series of  $f(x) = x, 0 < x < L$ .
31. Solve the integral equation  $y(t) = t + \int_0^t y(\tau) \sin(t - \tau) d\tau$ .
32. Using Simpson's rule evaluate the integral  $\int_1^2 x dx$  with  $n = 4$  and hence find an upper bound for the error incurred.
33. Apply improved Euler method in 3 steps to solve  $y' = y, y(0) = 1$  with  $h = 0.1$ . Also find the error occurred.

(6 x 5 = 30 marks)

**Part D***Answer any two questions.*

34. (a) Solve  $x y'' - 4xy' + 6y = 21x$  .

(b) Solve the initial value problem  $(D^2 + 4)y = -12 \sin 2x$ ,  $y(0) = 1.8$ ,  $y'(0) = 5$ .

35. Solve the integral equation  $y(t) = t + \int_0^t y(\tau) \sin(t - \tau) d\tau$ .

36. Find the **fourier** series of  $f(x) = \begin{cases} -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \frac{\pi}{4} & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$

**(2 x 10 = 20 marks)**