**D 73740** (Pages : 3) Name..............................

Reg. No............................

# FIRST SEMESTER B.C.A. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2014

(UG-CCSS)

Complementary Course

## CA 1C 02—DISCRETE MATHEMATICS

(2009 Admissions)

Time : Three Hours Maximum : 30 Weightage

### Part A (Objective Type Questions)

*Answer* **all** *twelve questions.*
*Each questions carries* $\frac{1}{4}$ *weightage.*

1. Give the inverse of the following implication "if it rains today, I will to College tomorrow."
2. Give an example for a tautology.
3. What is the numeric function corresponding to the generating function $\frac{1}{1-3z}$
4. Give an example for an abelian group.
5. Construct the truth table for the statement $p \ n \ p$.
6. Write the power set of A where A = $(a,$ b}.
7. Write down the Fibonacci sequence of numbers.
8. Given A = (2, 5, 6), B = (3, 4, 2). Evaluate A – B and B – A.

Fill in the blanks :

9. Any one-one mapping of a set S onto S is called a ——— of S.
10. A commutative ring with identity and without zero divisors is called an ———
11. The ——— quantifier was used to translate expressions such as "for all" and "every".
12. A group that has a generating set consists of a single element is known as a ——— group.

(12 x = 3 weightage)

### Part B (Short Answer Questions)

*Answer* **all** *nine questions.*
*Each question carries 1 weightage.*

13. Prove that if n is an integer and $n^3 + 5$ is odd, then n is even.
14. How many permutations of the letters ABCDEFGH contain the string ABC.
15. Let P $(x) : x$ is a person

    F $(x, y) : x$ is the father of y

    M $(x, y) : x$ is the mother of y

    Write the predicate "x is the father of the mother of y".

Turn over

16. What is the truth value of $\forall x p(x)$ where $p(x)$ is the statement '$x^2 < 10$' and the domain consist of the positive integers not exceeding 4.

17. Let n and $r$ be non-negative integers with $r$ S n. Then prove that $nC_r =$

18. Let Z be the set of integers. The operation * on $z$ is defined by $a*b = a+b+1$ for a, $b$ E Z. Find the identity element.

19. In how many ways we can point 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and the remaining ones are white.

20. Find the generating function of $8C_0, 8C_1, 8C_8$, 0, 0, ....

21. Show that the binary operation * defined on **(R, *)** where $x * y = \max(x, y)$ is associative.

(9 x 1 = 9 **weightage**)

**Part C (Short Essay Questions)**

*Answer any* **five** *questions.*
*Each question carries* 2 *weightage.*

22. Using the method of proof by cases. Prove that the triangle inequality which states that if $x$ and y are real numbers, then $|x|+|y| \geq ||x+y|$.

23. Prove that the fourth roots of unity, $1, -1, i, -i$ form an abelian multiplicative group.

24. What is the solution of the recurrence relation a, $= 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

25. Show that $nC_r + nC_{r-1} = n+1C_r$ where $n > r > 1$ and, n and $r$ are natural numbers.

26. Use generating functions to solve the recurrence relation $a_n = 3a_{n-1} + 2$, $a_0 = 1$.

27. Show that the set of all positive rational numbers forms an abelian group under the composition defined by $a * b = \frac{ab}{2}$.

28. Suppose **M** is a ring **of all 2 x 2 matrices with their elements as integers, the addition and multiplication of matrices being the two ring compositions. Then show that M is a ring with zero divisors.**

**(5 x 2 = 10** weightage)

**Part D (Essay Questions)**

***Answer any* two *questions.***
***Each question carries* 4** *weightage.*

**29. (a) Define a linear recurrence relation with constant coefficients.**

**(b) Solve the following difference equation by using the method of generating functions**

$a_r - \quad + 6a_{r-2} = 2^r + r, r \quad 2.$

30. (a) Give a prof by contradiction of the theorem "if $3n + 2$ is add then n is odd.

(b) Show that the following statements about the integer n are equivalent :—

$P_i$ : n is even, $P_2$ : $n - 1$ is odd $P_3$ : $n^2$ is even.

31. (a) Define the field and the integral domain.

(b) Prove that every field is an integral domain but the converse need not be true.

(2 x 4 = 8 weightage)