

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2011**

Computer Science

CSCI C 01—DISCRETE MATHEMATICS

(2010 admissions)

Time : Three Hours

Maximum Weightage : 36

**I. Answer all questions. 12 x 1 = 12 weightage.**

1. Write down the negation of the proposition for every number  $x$  there is a number  $y$  such that  $y < x$ .
2. What is the bit string corresponding to the difference of two sets. Use it to find  $A-B$  where  $A = \{2,3,5\}$  and  $B = \{3,5\}$ .
3. If  $A_n = [0, 1]$  where  $1 \in \mathbb{Z}$ , the set of integers find  $\bigcup_{n \in \mathbb{Z}} A_n$  and  $\bigcap_{n \in \mathbb{Z}} A_n$ .
4. Let  $A = \{1,2\}$  and  $B = \{a,b\}$ . Find all function  $f: A \rightarrow B$  and for each such function determine whether it is one to one, onto, both or neither.
5. Find the order of every element in the multiplicative group  $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ .
6. Draw the transition diagram of finite state automata  $A$  that accept even number of  $a$ 's over  $\{a,b\}$ .
7. Let  $V = \{S\}$ ,  $T = \{0,1\}$  and  $P = \{S \rightarrow 11S, S \rightarrow 0\}$  Find  $L(G)$ , give its verbal description.
8. Give example for a relation which is
  - a. Reflexive and transitive but not symmetric
  - b. Symmetric and transitive but not reflexive
9. 40 computer programmers interviewed for a job. 25 knew Java, 28 knew oracle, and seven knew neither language. How many knew both languages?
10. Determine whether the relation  $S = \{(a,b) : a \geq b\}$  on the set  $R$  of real numbers is an equivalence relation.
11. Assume the value of  $p \rightarrow q$  is false. Determine the value of  $(p \vee q) \rightarrow q$ .
12. Show that  $(p \vee q) \wedge (p \wedge q)$  is a contradiction.

**II. Answer any six questions. 6 x 2 = 12 weightage.**

13. Prove that  $A - (B \cap C) = (A - B) \cup (A - C)$ .
14. If the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \begin{cases} 3x-4 & x > 0 \\ -3x+2 & x \leq 0 \end{cases}$  determine
  - a.  $f(0), f(2/3), f(-2)$
  - b.  $f^{-1}(0), f^{-1}(2), f^{-1}(-7)$
15. If in a ring  $R$  with unity  $(xy)^{-1} = x^{-1}y^{-1}$  for all  $x, y \in R$  then  $R$  is commutative.

**Turn over**

16. Define the following with example
- a. Ring
  - b. Group
17. Show that  $D_{30}$  is finite Boolean algebra under partial order of divisibility.
18. Prove that  $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$  using
- a. Truth table
  - b. Algebra of proposition
19. Find the particular solution of difference equation
- $$a_{r+2} + a_{r+1} + a_r = r^2$$
20. Show that propositions  $q \rightarrow p$  and  $p \rightarrow q$  is not equivalent to  $p \rightarrow q$ .
21. Show that in any room of people who have been doing handshaking there will always be at least two people who have shaken hands the same number of times.

III. Answer any three questions. 3 x 4 = 12 weightage.

22. Find how many integers between 1 and 60 that are not divisible by 2 nor by 3 and nor by 5. Also determine the number of integers divisible by 5 not by 2, not by 3.
23. Prove that the fourth root of unity 1, -1, i, -i form an abelian multiplicative group.
24. Let  $R = \{(1,2), (2,3), (3,1)\}$  and  $A = \{1,2,3\}$  find the reflexive, symmetric and transitive closure  $R^*$ .
25. Solve the recurrence relation  $y_{k+1} - 5y_k = 5^k$  satisfying the initial condition  $y_0 = 3$ .
26. Define finite state machine with example. Design a finite state machine that performs serial addition.
27. Explain the following with example
- a: Lattices, distributive and complemented lattices.
  - b. Principle of duality.