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FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MAT 1C 03—REAL ANALYSIS—I

(2010 Admissions)

Time : Three Hours Maximum . 36 Weightage

Part A (Short Answer Questions) (1 —14)

Answer **all** *questions.*
Each question has 1 weightage.

1. Construct a bounded set of real numbers with exactly one limit point.

2. For x, y ***ER***^{1} let $d\,(x, y) = 1x^2 - y^2 l$. Is d a metric on R ? Justify your answer.

3. Let E be a non-empty set of real numbers which is bounded above and let y = sup E. Prove that

   y E E.

4. Is a finite set closed ? Justify your answer.

5. Prove that the limit of a function is unique.

6. Construct a function which has a simple discontinuity at every rational point.

7. Let f be a differentiable function on [a, b]. Prove that f is continuous on [a. b].

8. Let f be a continuous function and f 0 on $[a, b]$. If $\int_a f\,dx = 0$, then prove that $f(x) = 0$ for all

   x E $[a, b]$.

9. **Let f_p f_2 be bounded** functions and a be a monotonic increasing function on [a, b]. Prove that if f_2 are Reimann-Steiltjes integrable with respect to a on [a, b], then $f_1 + l_2$ is Reimann-Steiltjes integrable with respect to a on $[a, b]$.

10. Let f be a bounded function and a be a monotonic increasing function on $[a, b]$. If the partition P' is a refinement of the partition P of [a, b], then prove that.

$$L(I, f, \qquad L(P', f, a).$$

11. Let γ be defined on [0, 2π] *by* $7\,(t) = e^{'t}$. Prove that $'y$ is rectifiable.

Turn over

12. **Give an example of a convergent series of continuous functions with a discontinuous limit.**

13. **Prove that uniformly convergent sequence of bonded functions is uniformly bounded.**

14. **Define** equicontinuous **family of functions and give an example of it.**

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions* **(15 —24).**
Each question has weightage 2.

15. **Prove that the set of all integers is countable.**

16. **Prove that compact subsets of a metric space are closed.**

17. **Let** f **be a continuous real valued function on a metric space X. Prove that the set** $Z(f) = \{x \text{ E X } \; f(x) = 0\}$ **is a closed subset of X.**

18. **Let [x] denote the largest integer less than or equal to** x. **What type of discontinuities does the function** $[x]$ **have** ?

19. **If** f **is a real valued differentiable function on** (a, b). **If** $f'(x)$ **0 for all x E** (a, b), **then prove that** f **is monotonic increasing on** (a, b).

20. **Let** f **be a bounded function and a be a monotonic increasing function on** $[a, b]$. **Prove that if** f **is** Reimann-Steiltjes **integrable with respect to a on [a,** $b]$, **then** $|f$ **is** Reimann-Steiltjes **integrable with respect to a on [a,** $b]$ **and** $\left| \int_a f \, d\alpha \right| \int_a |f| \, d\alpha.$

21. **For** $1 < s < \infty$, **define** $\zeta(s) = \text{Y}_{n=1}$ —. **Prove that** $(s) = s \; \frac{\mathbf{x - [x]}}{x^{s+1}} dx$ **where** $[x]$ **denote the greatest integer less than or equal to** x.

22. **For what values of** x **does the series** $\frac{1}{1 + n^2 x}$ **converge absolutely.**

23. **Prove that the series** $\sum_{n=1} (1\text{-}1 \; \frac{x^{\wedge} \;\; n}{n^2}$ **converges uniformly in every bounded interval.**

24. Let K be compact, $f_n \in C(K)$ $n = 1, 2, 3, \ldots$ and let $\{f_n\}$ be pointwise bounded and equicontinuous on K. Prove that $\{f_n\}$ is uniformly bounded on K.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *from the following four questions* (25—28).
Each question has weightage 4.

25. (a) Prove that countable union of countable sets is countable.

    (b) Prove that the cantor set is perfect.

26. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

    (b) Prove that continuous image of a connected space is connected.

27. (a) State Taylor's theorem.

    (b) Let f be a bounded function, α be monotonic increasing function and α' is Reimann integrable on $[a, b]$. Prove that f is Reimann-Steiltjes integrable with respect to α on $[a, b]$ if and only if $f\alpha'$ is Reimann integrable on $[a, b]$.

28. Let γ be a curve on [a, b] and let γ' be continuous on $[a, b]$. Prove that γ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt.$$

(2 x 4 = 8 weightage)