D 72888 **(Pages : 4)** **Name..............................**

Reg. No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT IC 04—ODE AND CALCULUS OF VARIATIONS

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. **Determine the radius of convergence of the series** $1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 \cdot 6^2} + \cdots$

2. **Define regular singular point of a differential equation** $y'' + p(x)\,y' + Q(x)\,y = 0$ **and give an example of differential equation having regular singular point** $x = 1$.

3. **Find the indicial equation and its roots for the equation** $x^3 y'' + (\cos 2x - 1) y' + 2xy =$

4. **Evaluate** $\lim_{a \to \infty} F\left(a, a, \frac{1}{2}, \frac{-x^2}{4a^2}\right)$.

5. **Show that** $(x - t)\sum_{n=0} p_n(x)\,t^n = (1 - 2xt + t^2)\sum_{n=1} n p_n(x)\,t^{n-1}$, **where** $p_n(x)$ **is the** n^{th} **degree Legendre polynomial.**

6. **Define Gamma function and show that** $\overline{|n+1} =$ **n! for any integer n 0.**

7. **Show that** $J_{-m}(x) = (-1)^m J_m(x)$ **for any integer m 0.**

8. **Describe the-phase portrait of the system :** $\frac{dx}{dt} = -x, \frac{dy}{dx} = -y.$

Turn over

9. Determine whether the function $-2x^2+3xy-y^2$ is positive definite, negative definite or neither.

10. State sturm separation theorem.

11. Show that every non-trivial solution of $y''+(\sin^{\hat{}} x+1)\,y=0$ has an infinite number of positive zeros.

12. Show that the solutions of the initial value problem $y' = f\,y),\ y\,(x_0) = y_o$; where $f(x, y)$ is an arbitrary function defined and continuous in some neighbourhood of the point (x_U, y_o), are precisely the continuous solutions of the integral equation $y\,(x) = y_o + \int^{x} f\,(t, y\,(O)\,dt$

13. Prove that $f(x,y) = y$ Y2 does not satisfy a Lipschitz condition on the rectangle $0|$ and

14. What is the isoperimetric problem ?

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Find the general solution of $(1+x^2\ y''+2xy-2y=0$ in terms of power series in x.

16. Find the general solution of the differential equation :

$(x^2-x-6)y''+(5+3x)\quad +y=0$ near its singular point $x=3$.

17. Show that $P_n\,(x) = \dfrac{1}{2^n \cdot n!}\dfrac{d}{dx}\,(x^{9}-1)^{\prime\prime}$ satisfies the Legendre's equation :

$(1-x^{\hat{}})y''-2xy+n(n+1)y=0,$

where n is a non-negative integer.

18. Show that $\frac{2p}{x} \; _p(x) \quad J_{p-1}(x) + J_{p+1}(x)$.

19. If $f(x) = x^p$ for the interval $0 < x$ ~~show that its~~ Bessel series in the functions $J_p(\lambda_n x)$,

where the $\lambda'_n s$ are the positive zeros of $J_p(x)$ is $x^p - \frac{2}{n=1 \lambda_n J_{p+1}(\lambda_n} \; p(\lambda_n)$

20. Show that if the two solutions :

x = (t) and = x_2 (t)

= (t) = y_2 (t)

of the homogeneous system $\frac{dx}{dt} = (t)x + b_1(t)y$; $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ are linearly independent on [a, b], then $x = c_1x_1(t) + c_2x_2(t)$; $y = (t) + c_2 y_2(t)$ is the general solution of the system on [a, b].

21. Determine the nature and stability properties of the critical point (0, 0) for the system :

$$\frac{dx}{dt} = 4x - 2y, \frac{dy}{dt} = 5x + 2y$$

22. Let y (x) and z (x) be non-trivial solutions of $y'' + q(x)y = 0$ and $z'' + r(x)z = 0$, where $q(x)$ and $r(x)$ are positive functions such that $q(x) > r(x)$. Show that $y(x)$ vanishes at least once between any two successive zeros of $z(x)$.

23. Obtain Euler's differential equation for an extremal.

24. Show that the triangle with greatest area A for a given perimeter is equilateral.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 weightage.

25. Show that the differential equation :

$$x^2 y'' + xy + \left(x^2 - \frac{}{4}\right)y = 0$$

has two independent Frobenius series solutions and find them.

Turn over

26. State and prove the orthogonality property of Legendre polynomials.

27. Discuss the general solution of Bessel's equation.

28. Solve the initial value problem by Picard's method :

$$\left| \begin{array}{l} \dfrac{dy}{dx} = z, \quad y(0) = 1 \\ \dfrac{dz}{dx} = -y, \; z(0) = 0. \end{array} \right.$$

(2 x 4 = 8 weightage)