D 52973 (Pages : 3) Name

Reg. No.....................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MAT 10 03—REAL ANALYSIS—I

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions (1- 14)

Answer **all** *questions.*
Each question has 1 weightage.

1. Prove that every neighborhood is an open set.
2. In closure of a connected set connected ? Justify your answer.
3. Prove that a closed subset of a compact space is compact.
4. Let E be an infinite subset of a compact set K. Prove that E has a limit point in K.
5. Prove that the composition of two continuous functions is continuous.
6. Explain discontinuities of first and second kinds.
7. Let f (x) $= |x|^{-}$. Evaluate $f''(x)$ for all real x.
8. Let a be increasing on $[a, b]$ and continuous at $x_0 \in (a, b)$. If f (x_0) = 1 and $f(x) = 0$ for $x \neq x_0$, then prove that $\int_a^b f \, d\alpha = 0$.
9. Let f_1, f_2 be bounded functions and a be monotonic increasing function on [a, b]. Prove that if f_1, f_2 are Reimann-Steiltjes integrable with respect to a on [a, 1)] and f_1 $f_2(x)$ on [a, b], then prove that $\int_a^b f_1 \, dx \quad \int_a^b f_2 \, dx$.
10. Let f be Reimann integrable on [a, b] and for $a \; x$ let F (x) $f(t) \, dt$. Prove that F is continuous on [a, b].
11. Let $y \; [0, 1] \to \mathbb{R}^{-}$ be given by $y(x) = (2x, x^2 + 1)$. Prove that y is rectifiable.
12. If $\{f_n\}$ and $\{g_n\}$ converge uniformly E, then prove that $\{f_n + g_n\}$ coverge uniformly on E.

Turn over

13. Define **equicontinuous** family on functions and give an example of it.

14. Does a uniformly bounded sequence has a uniformly convergent subsequence ? Justify your answer.

(14 x 1 = 14 **weightage**)

Part B

Answer any **seven** *from the following ten questions.* (15 - 24)
Each question has weightage **2.**

15. Prove that countable union of countable sets is countable.

16. Prove that continuous image of a compact set is compact.

17. Let I = [0, 1] be a closed unit interval and let f be continuous mapping of I into I. Prove that $f(x) = x$ for atleast one $x \in I$.

18. Let [x] denote the largest integer less than *or* equal to x and let $(x) = x - [x]$. What type of discontinuities does the function (x) have ?

19. Let f be defined on [a, *b]*. If f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists, then prove that $f'(x) = 0$.

20. Show by an example that the L' Hospital rule need not true for vector valued functions.

21. For $1 < s < 00$, define $\zeta(s) = \sum_{n=1}^{00} \frac{1}{n^s}$. Prove that $\zeta(s) = s \frac{[x]}{x^{s+1}} dx$, where *[x]* denote the greatest integer less than *or* equal to x.

22. Let f be a bounded function and a be a monotonic increasing function on [a, ***b]***. Prove that if f is Reimann-Steiltjes integrable with respect to a on [a, ***b],*** then $|f|$ is Reimann-Steiltjes integrable with respect to a on [a, ***b]*** and $\left|\int_a^b d\, d\alpha\right| \quad \int_a^b |f| d\alpha.$

23. Let K be a compact metric space and let $f_n \in C(K)$ for n = 1, 2, 3, ... If f_n converges uniformly on K, then prove that $\{f_n$ **is** equicontinuous on K.

24. For n = 1, 2 . . . and x real let $I_n(x) = \frac{x}{1 + nx^2}$. Show that $\{f_n\}$ converges uniformly.

(**7 x 2** = 14 **weightage**)

Part C

Answer any two from the following four question **(25-28)**
Each question has weightage **4.**

25. (a) Let E be a non-compact set in $\mathbb{R}^1$. Prove that there exists a continuous function on E which is not bounded.

(b) Prove that monotonic functions have no discontinuities of the second kind.

26. Let f be a continuous mapping of compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

27. (a) If f is differentiable on $[a, b]$, then prove that f' cannot have any simple discontinuity on [a, b].

(b) Let f be a bounded function and a be a monotonic increasing function on [a, b]. Prove that is Reimann-Steiltjes integrable with respect to a on $[a, b]$ if any only if for every e > 0 there exists a partition P of [a, b] such that U (P, f, a) − L (P, f, a) < s.

28. (a) Prove that there exists a real continuous function on the real which is nowhere differentiable.

(b) State Stone-Weirstrass Theorem.

(2 x 4 = 8 weightage)