

D 52971

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Name.....

Reg. No.....

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014**

(CUCSS)

Mathematics

**MT IC 01—ALGEBRA—I**

Time : Three Hours

Maximum : 36 Weightage

**Part A**

*Answer all questions.*

*Each question carries 1 weightage.*

1. Define an isometry of  $\mathbb{R}^2$  and show that the product of two **isometries** is again an isometry.
2. Find the order of the element  $(3, 10, 9)$  in  $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$ .
3. Show that for word addition of binary words  $u$  and  $v$  of the same length, we have  $u + v = u - v$ .
4. Let  $H$  and  $K$  be subgroups of a group  $G$ . Give an example showing that we may have  $H \cong K$  while  $G/H$  is not isomorphic to  $G/K$ .
5. Define solvable group and give one example of it.
6. Let  $G$  be a group of order  $p^n$  and let  $X$  be a finite  $G$ -set. Show that  $|X| \equiv |X_G| \pmod{p}$ .
7. Obtain the class equation of  $S_3$ .
8. Show that no group of order  $p^r$  for  $r > 1$  is simple, where  $p$  is a prime.
9. How many different **homomorphisms** are there of a free group of rank 2 onto  $\mathbb{Z}_6$ ?
10. Show that  $(x, y : y^2 x = y, yx^{-1}y = x)$  is a presentation of the trivial group of one element.
11. Consider the evaluation **homomorphism**  $\phi_5 : \mathbb{Q}[x] \rightarrow \mathbb{R}$ . Find six elements in the Kernel of  $\phi_5$ .
12. Find all generators of the cyclic multiplicative group of units of the field  $\mathbb{Z}_7$ .
13. Show that the fields  $\mathbb{R}$  and  $\mathbb{C}$  are not isomorphic.
14. Give an example to show that a factor ring of an integral domain may not be a field.

(14 x 1 = 14 weightage)

**Turn over**

**Part B**

Answer any **seven** questions.

Each question carries 2 weightage.

15. Show that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.
16. Let  $H$  be a normal subgroup of a group  $G$ , and let  $m = (G : H)$ . Show that  $a^m \in H$  for every  $a \in G$ .
17. Find the center of  $S_3 \times Z_5$ .
18. Give isomorphic refinements of the two series :  
 $\{0\} < (18) < (3) < Z_{72}$  and  $\{0\} < (24) < (12) < Z_{72}$ .
19. Let  $X$  be a  $G$ -set. Show that  $G_x$  is a subgroup of  $G$  for each  $x \in X$ .
20. Find the number of distinguishable ways the edges of a square of cardboard can be painted if six colours of paint are available, assuming no colour is used more than once and the same color can be used on any number of edges.
21. Show that there are no simple groups of order  $p^r m$ , where  $p$  is a prime and  $m < p$ .
22. Write all polynomials of degree  $< 2$  in  $Z_2[x]$ .
23. Show that a non-zero polynomial  $f(x) \in F[x]$  of degree  $n$  can have at most  $n$  zeros in a field  $F$ .
24. Let  $R$  be a commutative ring and let  $a \in R$ . Show that  $I_a = \{x \in R : ax = 0\}$  is an ideal of  $R$ .

(7 x 2 = 14 weightage)

**Part C**

Answer any **two** questions.

Each question carries 4 weightage.

25. Let  $H$  be a subgroup of a group  $G$ . Show that left coset multiplication is well defined by the equation  $(aH)(bH) = (ab)H$  iff left and right cosets coincide.
26. Show that if  $N$  is a normal subgroup of a group  $G$  and if  $H$  is any subgroup of  $G$ , then  $H \vee N = HN = NH$  and  $\frac{HN}{N} \sim \frac{H}{(H \cap N)}$ .
27. Let  $P_1$  and  $P_2$  be **syllow**  $p$ -subgroups of a finite group  $G$ . Show that  $P_1$  and  $P_2$  are conjugate. Verify this theorem for  $S_4$  with  $p = 3$ .
28. Determine all groups of order 8 upto isomorphism.

(2 x 4 = 8 weightage)