D 52972 (Pages : 3) Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MT IC 02—LINEAR ALGEBRA

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that for a, $_3$ in a vector space V, -(a +13) = -a + (−β).
2. Show that U = {(x, 2x) : x e RI is a subspace of $\mathbb{R}^{\hat{}}$.
3. Verify whether the set of all 3 x 3 diagonal matrices span the space of all 3 x 3 matrices.
4. Find the dimension of the space of all upper triangular 2 x 2 matrices over R .
5. Find the Co-ordinate vector of (1, -1, 2) E R^3 w.r.t. the ordered basis {(1, 0, 1), (1, 0, 0), (1, 1, 0)}.
6. Let T : $R^2 \to R^3$ be defined by $T(x, y) = (x + y, xy, y)$. Verify whether T is linear.
7. Let W = span {(1, 0, 1), (0, 0, 1)}. Let $f: R^3 \to R$ be defined by :

   $f(x, y, z) = y$. Verify whether f E W°.
8. Find the characteristic polynomial of $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.
9. Find all characteristic values of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.
10. Let W = Span ((1, 2, 1)} and T : R^3 R^3 be defined by $T(x, y, z) = (x + z, 2y, 2z)$. Verify whether W is an invariant subspace of T.
11. Let W_1 = span {(1, 1, 1), (1, 1, 2)), and W_2 = span {(2, 2, 3)). Verify whether $W_1 + W_2$ is a direct sum.
12. Verify whether T R^2 defined by T(x, y) = $\frac{x+y}{2}$ $\frac{x}{2}$ y is a projection.

Turn over

13. Verify whether **(1, 1), (1,** -1) are orthogonal in R^2.

14. Prove that if E is an orthogonal projection of V onto W then $\mathcal{I}m(1-E) = W^{\perp}$.

(14 x **1** = 14 weightage)

Part B (Paragraph Type)

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Let V be a vector space over a field F and c 0. Prove that if ca = 0 for some a E V, then a = 0.

16. Verify whether $S = \{(x, y, x+y+1) : x, y \in \mathbb{R}\}$ is a subspace of

17. Show that the row space of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix}$ is **R3.**

18. Show by an example that for non-zero subspaces $\mathbf{W_1}$, $\mathbf{W_2}$ of a vector space V, dim $(W_1 + W_2)$ can be equal to dim W_1.

19. Find the matrix of the transformation T $R^3 \to R^3$ defined by $T(x, y, z) = (x+y, y+z, y)$ w.r.t. the ordered basis ((0, 1, 1), (1, 1, 0), (1, 0, 0)1.

20. Let $V = R^2$ and $B = \{(1, 0), (0, 1)\}$. Find the dual basis of V^* corresponding to B.

21. Let T be an invertible linear operator and $c \neq 0$ be a characteristic value: **of** T. Prove that $\frac{1}{c}$ is a characteristic value of T^{-1}.

22. Express **le** as a direct sum W_1 $\mathbf{W_2}$, where W_1 = span ((1, 1, 1), (1, 2, 1)).

23. Let T be a linear operator on V and let $V = W_1 \oplus \mathbf{W_2}$, where W_1 and $\mathbf{W_2}$ are invariant under T. Prove that $\det(T) = \det(T_1) . \det(T_2)$ where T_i is the restriction of T to W_i for $i = 1, 2$.

24. Verify whether (x y) defined by $(x \mid y) = x1 + y_1$ for $x = (x_1, x_2)$, $y = (y_1, y_2)$ is an inner product on R^2.

(7 x 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** *questions.*
Each question carries weightage 4.

25. Define Co-ordinate vector of an element a in a finite dimensional vector space V, over a field F. Let B be an ordered basis of V and P be an n x n invertible matrix over F where $n = \dim V$. Prove that there exists a unique ordered basis B' of V such that $[\alpha]_B = P[\alpha]_{B'}$ for all a E V

26. (a) Let $\{a_1, \alpha_2, \ldots, \alpha_n$ I be a basis of a vector space V and β_1, P_2 ,β_n be elements of V. Prove that there exists a unique linear operator T on V such that $T(\alpha_i) = \beta_i$ for every i.

(b) Find all linear operators on the one-dimensional space III over R .

27. Define the transpose T^t of a linear transformation T. Prove that for linear transformations of finite dimensional spaces, rank (T) = rank (Ti).

28. Let V be a finite dimensional space and $W_1, W_2, \ldots, W_k$ be subspace of V and let $V = W_1 + \ldots + W_k$. Prove that the following are equivalent

(a) $W_1, W_2, \ldots, W_k$ are independent.

(b) If B_i is a basis of W_i for $i = 1,.., k$ then $B = B_1 \cup B2 \text{ v} \ldots \text{ v } B_k$ is a basis for V.

(2 x 4 = 8 weightage)