D 52974 **(Pages : 4)** **Name...**

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MT IC 04—ODE AND CALCULUS OF VARIATIONS

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. **Determine the radius of convergence of the series** $\sum_{n=1} \frac{p(p-1)(p-2)\ldots(p-n+1)}{n} x$

2. **Locate and classify the singular points on the x-axis of the differential equation**

   $x^3 (x-1) y'' - 2(x-1) y' + 3xy = 0.$

3. **The equation** $y'' + p \quad {}_4 \; y = 0$, **where** p **is a constant has a series solution of the form**

   $y = \quad a_n x^n$. **Show that the coefficients** a_n **are related by the formula**

   $(n+1)(n+2) a_{n+2} + \left(p + \frac{1}{2}\right) a_n - {}_4 \; a_{n-2} = 0.$

4. **Determine the nature of the point** $x = 00$ **for Legendre's equation** $(1 - x^2) y'' - 2xy' + p(p+1) y = 0.$

5. **Verify that** $p_n(-1) = (-1)^n$ **where** $p(x)$ **is the nth degree Legendre polynomial.**

6. **Write Bessel's equation of order two and show that** $x = 0$ **is a regular singular point of it.**

7. **Show that** $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$

8. **Describe the phase portrait of the system** $\frac{dx}{dt} = x, \ \frac{dy}{dt} = 0.$

Turn over

9. **Determine whether the function $-x^2 - 4xy - 5y^2$ is positive definite, negative definite, or neither.**

10. **Show that every non-trivial solution of $y'' + (\sin^{-} x + 1)\,y = 0$ has an infinite number of positive zeros.**

11. **State Picard's theorem.**

12. **Show that $f(x, y) = xy^{-}$ does not satisfy a Lipschitz condition on any strip $a\ x\ \ b$ and $-a < y < a$.**

13. **Find the extremal for** $\Gamma \ \overline{\ (1)^2\ }\ dx$

14. **What is the isoperimetric problem.**

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Express $\sin^{-}$ x in the form of a power series $\sum a_n x^n$ **by solving $y' = (1 - x^2)$** $y(0) = 0$; in two ways.

16. Find the general solution $y'' + xy' + y = 0$.

17. **Show that the solutions of the equation $(1 - x^2)y''\ \ 2xy' + n(n+1)y = 0$,** where n is a non-negative integer, bounded near $x = 1$ are precisely constant multiples of $F\left[-n, n+1, 1, \frac{1}{2}(1 - x)\right]$.

18. Find the first three terms of the Legendre series of

$$f(x) = \begin{cases} 0 & if\ -1 \leq x < 0 \\ x & if\ 0 \leq x \end{cases}$$

Show that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$ and that between any two positive zeros of $J_1(x)$ there is a zero of $J_0(x)$.

19. Find the critical points and the differential equation of the paths of the non-linear system :

$$\frac{dx}{dt} \; ^{\cdot\prime} \; \overset{2}{+} 1), \quad \frac{dy}{dt} = 2xv^2$$

20. If $f(x) = \begin{cases} 1 & , \; 0 \le x < ½ \\ ^1/_2 & , \; x = ^1/_2 \\ 0 & , \; ½ < x \le 1, \end{cases}$

then show that $f(x) = \sum_{n=1}^{\infty} \frac{J_1\left(\frac{\hat{}}{2}\right)}{\lambda_n \; J_1(\lambda_n)^2} J_o(\lambda, \; x)$, where the $\lambda_n' s$ are the positive zeros of $J_0(x)$.

21. Verify that (0, 0) is a simple critical point of the system

$\frac{dx}{dy} = x + y - 2xy, \; \frac{dy}{dt} = -2x + y + 3y^2$ and determine its nature and stability properties.

22. State and prove sturm separation theorem.

23. Find the exact solution of the initial value problem $y' = y^2$, y (0) =1 : starting with $y_0(x) = 1$, apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$, and compare these results with the exact solution.

24. Find the curve of fixed length L that joins the points (0, 0) and (1, 0), lies above the x-axis and encloses the maximum area between itself and the x-axis.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*

Each question carries 4 *weightage.*

25. Find two independent Frobenius series solutions of the equation $x^2 y'' - x^2 y' + (x^2 - 2) y = 0$.

26. Determine the general solution of the hyper geometric equation

$$x(1-x)y'' + [c - (a + b + 1)x]y' - aby = 0$$

Turn over

27. Find the general solution of the system $\frac{dx}{dt} = 7x + 6y, \quad \frac{dy}{dt} = 2x + 6y.$

28. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition

If $(x, y_1) - f(x, Y2)1 \quad k|y_1 \quad y_2|$

on a strip defined by $a \leq x \leq b$ and $-a < y < co$. If (x_U, y_U) is any point of the strip, then the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a < x < b$.

(2 x 4 = 8 **weightage**)