D 52975 **(Pages : 3)** **Name..........**

Reg. No

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MT IC 05—DISCRETE MATHEMATICS

(2010 Admissions)

Time : Three Hours **Maximum :** 36 Weightage

Part A (Short Answer Questions) (1-14)

Answer **all** *questions.*
Each question has 1 weightage.

1. Let (X, and (Y, $_2$) be partially ordered sets and let Z = X x Y. Define a relation $<$ on Z as follows :

   $y_1) < (x_2, y_2) \Leftrightarrow$ x_2 and $y_1 <_2 y_2$.

   Prove that $<$ is a partial order on Z.

2. Let X be the set of positive integers which divide 30. Define a relation $<$ on X by x y if and only if x I y . Draw the Hasse diagram of this relation.

3. Let (X, +, . ,) be a Boolean algebra. Prove that (x')' = x for all x E X.

4. Prove that $x_1 x_2 + x_3$ is symmetric with respect to x_1 and x_2.

5. Find the girth of the graph K_5.

6. Define Eulerian graph. Determine the values of m and n such that $K_{m,n}$ is Eulerian.

7. If G is a simple graph, then prove that dim G 3 implies dim G _ 3.

8. Let G be a graph with connectivity 4. Is G 2-connected ? Justify your answer.

9. Prove that $K_{3,3}$ cannot be drawn without crossing.

10. Let $l(F_i)$ denotes the length of face F_i in a plane graph G, then prove that $2e(G) = \sum l\ (F_1)$,

    where $e(G)$ is the numb r of edges in G.

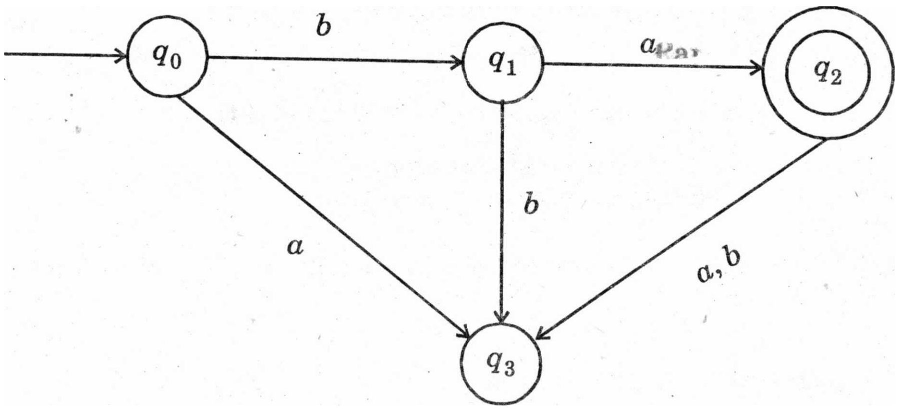
Turn over

11. Find a grammar for —— $= \{a, b\}$ that generates the set of all strings with exactly one a.

12. Define a regular language and give an example of it.

13. Show that if L is regular, then so is L – {X).

14. Find the set of strings accepted by the following deterministic acceptor.



(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions* (15-24).
Each question has weightage 2.

15. Let (X, +, . ,) be a finite Boolean algebra. Prove that every element of X can be uniquely expressed as a sum of atoms.

16. Prove that in a distributive, bounded lattice, the complements are unique, whenever they exist.

17. Prove that every u, v -walk contains a u, v -path.

18. Prove that the number of odd degree vertices in a given graph is even.

19. Prove that an edge is a cut edge if and only if it belongs to no cycle.

20. Prove that a tree with n vertices has n 1 edges.

21. If G is a simple graph, then prove that

$k(G) \quad (G) < \delta (G)$.

22. Let G be a simple planar graph with atleast three vertices. If G is triangle-free, then prove that $e(G)$ 5 $n(G) - 4$, where e (G) and n (G) denote the number of edges and vertices in G respectively.

23. Find a grammar that generates the language fan b^{en} : $n > 0\}$.

24. Let E = $\{a, b, c\}$. Construct a deterministic finite acceptor that accepts that language $a\Sigma * b$.

(7 x 2 = 14 weightage)

Part C

Answer any two from the following four questions (25-28).
Each question has weightage 4.

25. (a) Prove that every finite Boolean algebra is isomorphic to a powerset Boolean algebra.

(b) Write the Boolean function

$$f(x_1, x_2, x_3) = (x_1 + \quad x_3' + x_2x_1 \quad + x_1'x_3)$$

in their disjunctive normal form.

26. Prove that the complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \leq 2^k$.

27. (a) Prove that every connected graph contains a spanning tree.

(b) If a connected plane graph G has exactly n vertices, e edges and f faces, then prove that $n - e + f = 2$.

28. (a) Let (Q, E, 6, q_0, F) be a deterministic finite acceptor accepting the language L. Prove that (Q, E, 6, go, $Q - F$) accepts the language $E* - L$.

(b) Let G_M be the transition graph of some deterministic finite acceptor M. If $L(M)$ is infinite, then prove that G_M must have at least one cycle for which there is a path from the initial vertex to some vertex in the cycle.

(2 x 4 = 8 weightage),