D 92951 **(Pages : 2)** **Name..**

Reg. No......................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

Time : Three Hours **Maximum : 36 Weightage**

Part A (Short Answer Type)

Answer **all** *the questions.*
Each question carries weightage 1.

1. **Write an example for one dimensional vector space.**
2. **Show that the null space of a linear transformation is a subspace of the domain space.**
3. **Define basis for a vector space.**
4. **For any *two* real numbers x and y, lets $S = \{(x, y) : 2x + 3y + 4 = 0\}$. Is the set S a subspace of the two dimensional Euclidean space ? Justify your claim.**
5. **Define $T : R^2 \to R^2$ by the rule $T(x, y) = (1 + x, y)$. Is T a linear transformation ? Justify your claim.**
6. **Define linear functional and give an example for a linear functional.**
7. **Define hyperspace of a vector space.**
8. **If S is a subset of a finite dimensional vector space V, with usual notation prove that $(S^0)^0$ is the subspace spanned by S.**
9. **If A and B are two matrices that are similar, what can we say about their characteristic polynomials ? Justify your claim.**
10. **Prove that R is a subspace of the inner product space R^3 with usual inner product.**
11. **Write the relation between linear transformation and matrices in finite dimensional vector spaces.**
12. **Define projection map. Show that projection map is diagonalizable.**
13. **When do we say that a matrix A is the direct sum of the n matrices $A_1, A_2,$**
14. **Show that an orthogonal set of non-zero vectors is linearly independent.**

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Prove that in a finite dimensional vector space, every non-empty linearly independent set of vectors is a part of a basis.

16. Let V and W be vector spaces over the field F and T be a linear transformation fro V to W. If T is invertible, then show that the inverse function T^{-1} is a linear transformation from W onto V.

17. Prove that every n-dimensional vector space over the field F is isomorphic to the space

18. Let V and W be vector spaces over the field F and T be a linear transformation from V into W. Suppose that V is finite-dimensional. Then prove that rank (T) + nullity (T) = dim V.

19. Let $B = \{a_1, a_2, a_3\}$ be the ordered basis for R^3 consisting of $a_1 = (1, 0 -1)$, $a_2 = (1, 1, 1)$, $a_3 = (1, 0, 0)$. What are the co-ordinates of a vector *(a, b, c)* in the ordered basis B ?

20. Let V be a finite dimensional vector space over the field F. Then prove that each basis of V* is the dual of some basis of V.

21. Let V be a vector spaces over the field F. If L is a linear functional on the dual space V* of V, prove that there is a unique vector a in V such that L (f) = $f(\quad)$.

22. Let V and W he finite dimensional vector spaces over the field F. Prove that V and W are isomorphic if and only if dim V = dim W.

23. State and prove the Cauchy-Schwarz inequality.

24. Prove that every finite dimensional inner product space has an orthonormal basis.

(7 x 2 = **14** weightage)

Part C (Essay Type)

Answer any **two** *questions.*
Each question carries weightage **4.**

25. Let V be an m-dimensional vector space over the field F and W be an n-dimensional vector spaces over F. Then with usual assumptions prove that the space **L (V, W)** is a finite-dimensional vector space of dimension *mn*.

26. State and prove the Cayley - Hamilton theorem for a linear operator on a finite dimensional vector space.

27. Define characteristic polynomial *f (x)* and minimal polynomial *p (x)* of a linear operator on an n-dimensional vector space V. Show that *f (x)* and *p (x)* have the same roots except for multiplicities.

28. Check whether the operator T given by T (x, y, z) = (x + y + z, 2y + z, 3z) is diagonalizable or not.

(2 x 4 = 8 weightage)