D 92952 **(Pages : 3)** Name.................

Reg. No...............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 1C 03—REAL ANALYSIS—I

Time : Three Hours Maximum : 36 Weightag

Part A

Short answer questions **(1-14)**
Answer **all** *questions.*
Each question has 1 weightage.

1. Prove that balls are convex.
2. Give an example of a perfect set which is bounded.
3. Prove that $d((x_1, y_1), (x_2, y_2)) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ is a metric on R^2.
4. Let E' be the set of limits points of a set E. Prove that E' is closed.
5. Does there exists a continuous real valued function f on [0, 11 such that f is not uniform continuous ? Justify your answer.
6. Let $f \quad \to \mathbb{R}$ be a function defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer *le* than or equal to x. What type of discontinuities does the function f have ?
7. Prove that differentiable functions are continuous.
8. Let f be a differentiable function on [a, b] such that $f'(x) = 0$ for all $x \in (a, b)$. Prove that constant.
9. Evaluate $\lim_{x} \dfrac{\sin\left(\frac{\pi}{2}x\right)}{x - 2}$
10. Let f be a function on $[a, b]$ such that $|f|$ is Riemann integrable on $[a, b]$ ($|f| \in \mathcal{R}$ on $f \in \mathcal{R}$ on $[a, b]$? Justify your answer.
11. Let $f \in \mathcal{R}$ on $[a, b]$ and for $a \leq x \quad b$ let $F(x) = \int_a^x f(t)\,dt$. Prove that F is continu
12. Let y be a curve in the complex plane, defined on $[0, 2\pi]$ by $y(t) = e^{it}$. Find the lens
13. Define equicontinuity and give an example of it.
14. State Stone-Weierstrass theorem.

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Part B

Answer any seven *from the following ten questions* (15-24)
Each question has weightage 2.

15. Prove that every infinite subset of a countable set is countable.

16. Prove that finite intersection of open sets is open. Is arbitrary intersection of open sets open ? Justify your answer.

17. If E is an infinite subset of a compact set K, then prove that E has a limit point in K.

18. Prove that monotonic functions have no discontinuities of the second kind.

19. Let f be a differentiable function on (a, b). If $f(x) \leq 0$ for all $x \in (a, b)$, then prove that f is monotonically decreasing.

20. Let a be a monotonically increasing function and f be a bounded function on [a, b]. Prove that

$f\ d\alpha \quad f\ d\alpha.$

21. For $1 < s < \infty$, let $\zeta(s) = \frac{1}{n^s}$ n o n Prove that $(s) = s \int_{\lambda}^{\infty} \frac{[x}{}$

where [x] denotes the largest integer less than or equal to x.

Let $\{f_n\}$ be a sequence of real valued Riemann integrable functions on a set E such that $f_n \to f$ as $n \to$ ɔ. Is it true that $J f_n$ f ? Justify your answer.

Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

Let $\{f_n\}$ be a sequence of equicontinuous functions on a compact set K. If $\{f_n\}$ converges pointwise n K, then prove that f_n converges uniformly on K.

(7 x 2 = 14 weightage)

Part C

Answer any two from the following four questions (25-28)
Each question has weightage 4.

that a subset E of is connected if and only if it satisfies the following : If x, y $\in$ E and y, then $z \in$ E.

a bounded real function on [a, b] and a be monotonically increasing on [a, b]. If f has navy discontinuities on *[a, b]* and a is continuous at every point at which f is discontinuous, re that f Riemann-Stieltjes integrable with respect to a on [a, b].

27. Prove that there exists a real continuous function on the the real line which is nowhere differentiable.

28. Let $\{f_n\}$ be a sequence of continuous functions on a compact set K. If $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that

(a) $\{f_n\}$ is uniformly bounded on K.

(b) $f_n\}$ contains a uniformly convergent subsequence.

(2 x 4 = 8 weightage)