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Name..............................

Reg. No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 1C 04—ODE AND CALCULUS OF VARIATIONS

Time : Three Hours

Maximum : 36 Weightage

Part A

*Answer **all** questions.*
Each question carries 1 weightage.

1. Find a power series solution of the form $\sum a_n x$ of the differential equation $y' + y = 1$.

2: Determine the nature of the point $x = -1$ for the equation $x^2 (x^2 - 1)^2 y'' - 2(x - 1) y' + 3xy = 0$.

3. Find the indicial equation and its roots of the equation $x^3 \quad + (\cos 2x - 1) \quad + 2xy = \quad$.

4. Find the general solution of the equation $(2x^2 + 2x) y'' + (1 + 5x) y' + y = 0$ near the singular point $x = 0$.

5. Find the first two terms of the Legendre series of the function

$$f(x) = \begin{cases} 0 \text{ if } -1 5\_ x < 0 \\ x \text{ if } \quad 0 \le x \le 1 \end{cases}$$

6. Prove that for an integer $m \quad 0$, $J_{-m}(x)$ and $J_m(x)$ are linearly dependent.

7.. Prove that $\frac{d}{dx} \quad J \quad (x)\rfloor = x^p J_{p-1}(x)$.

8. Describe the phase portrait of the system $\frac{dx}{dt} = -x, \frac{dy}{dt} = -y$.

9. Find the critical points of the non-linear system :

$$\frac{dx}{dt} = -x, \quad \frac{dy}{dt} = 2x^2 \quad y^2.$$

Turn over

10. Show that (0, 0) is an asymptotically stable critical point of the system :

$$\frac{dx}{dt} \quad 3x3 \quad y, \frac{dy}{dt} \quad x^5 - 2y^3$$

11. Find the normal form of Bessel's equation $x^2 y''$ $xy' +$ (x2 p2) y o.

12. State sturm comparison theorem.

13. Show that $f(x, y) = xy$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \quad y \le d$.

14. Find the extremal for the integral $\int_x \left[y^2 \quad \right] dx$.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Find the series solution y (x) of the. differential equation y" + $- xy = 0$ satisfying the condition y (0) = 0, $y'(0) = 1$.

16. Find the only Frobenins series solution of the equation $x^2 \quad - 3xy' + (4x + 4) y = 0$.

17. Determine all the regular singular points of the hyper geometric equation x(1— x) y" + $[c - (a + b + 1) x] \quad - aby = 0$.

18. Show that if $p_n(x)$ is defined by $p_n(x)$ n1 $(x - 1)$, then p_n (x) satisfies the Legendre's equation $(1 - x^2)$ $'' - 2xy' + n(n + 1) y = 0$, where n is a non-negative integer.

19. Show that between any *two* positive zeros of J_0 (x) there is a zero of $J_1(x)$ and that between any *two* positive zeros of $J_1(x)$ there is a zero of J_0 (x).

20. If f (x) is defined by $f(x) = \begin{cases} & \text{if} \quad 0 \le x < \frac{1}{2} \\ & \text{if} \quad x = \frac{1}{2} \\ 0 & \text{if} \quad \frac{1}{2} < x \le 1 \end{cases}$, show that $f(x) = \sum_{n=1} \lambda_n J_1(\lambda \cdot J_0(\lambda_n x)$

where the λ_n's are the positive zeros of J_0 (x).

21. Determine the nature and stability properties of the critical point (0, 0) for the system :

$$\frac{dx}{dt} = 4x - 2y, \frac{dy}{dt} = 5x + 2y.$$

22. Show that if $q(x) < 0$, and u (x) is a nontrivial solution of $+ q(x)\, u = 0$, then u (x) has at most one zero.

23. Using the method of Lagrange multipliers, show that the triangle with greatest area A for a given perimeter is equilateral.

24. Find the exact solution of the initial value problem y' = 2x (1 + y), y(0) = 0. Starting with y_0 (x) = 0, apply Picard's method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and compare these results with the exact solution.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Discuss the general solution of the huper geometric equation

$$x(1-x)\, y'' + [c - (a + b + 1)\, x]\, y' - aby = 0$$ near the singular point $x = 0$.

26. State and prove the orthogonality property of the Legendre polynomials.

27. Find the general solution of the system :

$$\frac{dx}{dt} = 7x + 6y, \frac{dy}{dt} - 2x + 6y.$$

28. Obtain Euler's differential equation for an extremal.

(2 x 4 = 8 weightage)