D 92950 (Pages : 3) Name..

Reg. No......................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT IC 01—ALGEBRA-I

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1 **Give an example of an isometry of the plane which leaves the X – axis fixed.**

2. **Verify whether the direct product of two abelian groups is abelian.**

3. **Find the order of the element (2, 2) in** Z3 x Z4.

4. **Let** u = **1100101 and v = 1011101 be binary codes. Find** $d\,(u,$ **v).**

5. **Let G be the symmetric group** S3 **and H a subgroup of order 3 in** S3. **List the elements in** G/H.

6. **Verify whether the series (0) 5_ < 5 > S** Z15 **and (0) 5_ <3 > <** Z15 **are isomorphic.**

7. **Let H be a subgroup of a group G and G be an H – set defined by** $h * g = hg$. **Find the orbit of** e **where** e **is the identity of G.**

Find all sylow **2-subgroups of** S3.

9. **Find the reduced word corresponding to** $a_1\hat{a}_2a_2^{-1}\,\hat{a}_3a_3^{-1}$.

10. **List all the elements of the group whose presentation is (a,** b $= 1, b^2 = 1, ab = ba)$.

11. **Verify whether** x **– 2 is a factor of** $\mathbf{x^4 - 3x^2 + 2x + 1}$ **in** $\mathbb{Q}\,[x]$.

12. **Verify whether** $\mathbf{x^3 + 3x^2} + x + 1$ **is irreducible in** Z5 $[X]$.

Turn over

13. Find the multiplicative inverse of $i + j$ in the skew field of quaternions.

14. Verify whether the fields R and C are isomorphic.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Prove that if *m and* n are relatively prime then $\mathbb{Z}_m$ x $\mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$.

16. Prove that cyclic groups of order p^n where p is a prime are indecomposable.

17. Let H be a normal subgroup of a group G. Show that 4) : G $\rightarrow$ G/H defined by 4) *(x)* = xH is a homomorphism.

18. Let G = H x K be a direct product of groups. Let $\text{H} = \{(h,e) : h \in \text{H}\}$. Show that G/$\overline{\text{H}}$ is isomorphic to K.

19. Let X be a G – set. Show that Xis the disjoint union of its orbits.

20. Let N and H be normal subgroups of a group G. Show that NH is a normal subgroup of G.

21. Prove that every finite p-group is solvable.

22. Let R[x] be ring of polynomials over a ring R. Show that R[x] is commutative if R is commutative.

23. Show that $x^2 = 3$ has no solutions in rational numbers.

24. Let $p(x)$ be irreducible in F[x] where F is a field and $r(x)$, $s(x)$ E **F** [x]. Show that if *p (x)* divides $r(x)$ $s(x)$ then $p(x)$ divides $r(x)$ or $p(x)$ divides $s(x)$.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Let H be a normal subgroup of a group G. Describe the factor group GM. Show that if : G -5 G' is an onto homomorphism then G/K is isomorphic to G' where K is the kernel of 4).

26. Let X be a G – set. Define the orbit Gx and the isotropy group G_x *of x* E X. Prove that

$|Gx| = (G : G_x)$.

27. Describe the free group generated by a set A. Show that every group is a homomorphic image of a free group.

28. Show that every non-constant polynomial *f (x)* E **F** *Ix]* can be factored into a product of irreducible polynomials in F $[x]$.

(2 x 4 = 8 weightage)