) 33331 **(Pages : 3)** Name

Reg. No. . . . .

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2010 Admissions)

rime : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Define isometry of R^2 and give an examples of it.
2. Find all proper non-trivial subgroups of $\mathbf{Z}_2$ **x** $\mathbf{Z}_2$ **x** $\mathbf{Z}_2$.
3. Show that for two binary words of the same length, we have $d(u, v) = wt(u - v)$.
4. Let n be a positive integer and R be the group of all real numbers under addition. If

   $n\mathrm{R} = \{nr : r\ e\ \mathbf{R}\}$ in a subgroup of R, compute the factor group $\frac{\;}{n\mathrm{R}}$.
5. Determine the center of $\mathbf{A}_5$.
6. Define a p-group and give one example of it.
7. Obtain the class equation of a finite group G.
8. Show that every group of prime power order is solvable.
9. How many different homomorphism are there of a free group of rank 2 into $\mathbf{S}_3$?
10. Determine the Kernel of the evaluation homomorphism $\phi_i : Q[x] \to C$.
11. If F is a field and $a \neq 0$ in a zero of $f(x) = a_0 + a_1 x + \ldots + a_n x^n$ F [x], show that $\frac{\;}{a}$ in a zero of

    $a_n + a_{n-1} x + \quad + a_0 x$
12. **Let G** $= \{e, a, b\}$ be a cyclic group of order 3 with identity element e. Write the element $(2e + 3a + 0b)(4e + 2a + 3b)$ in the group algebra $\mathbf{Z}_5$ (G) in the form $re + sa + tb$ for $r, s, t,$ **c** $\mathbf{Z}_5$.

Turn over

13. **State division algorithm for F $[x]$, where F is a field.**

14. **Give an example to show that a factor ring of an integral domain may have divisors of zero.**

(14 x 1 = 14 weightage)

Part B

Answer **any seven** *questions.*
Each question carries 2 *weightage.*

15. **If m divides the order of a finite abelian group G, then show that G has a subgroup of order m.**

16. **Show that if a finite group G has exactly one subgroup H of a given order then H is a normal subgroup of G.**

17. **Show that if G has a composition series, and if N is a proper normal subgroup of G, then there exists a composition series containing N.**

18. **Let X be a G-set and let x c X. Show that I GxI = (G : G_x).**

19. **Find the number of orbits in {1, 2, 3, 4, 5, 6, 7, 8) under the cyclic subgroups ((1, 3, 6)) of S_8.**

20. **Show that every group of order $(35)^3$ has a normal subgroup of order 125.**

21. **Prove that if D is an integral domain, then D *[x]* is an integral domain.**

22. **Show that the multiplicative group of all non-zero elements of a finite field is cyclic.**

23. **If G =*{e)*, the group of one element, show that R (G) is isomorphic to R for any ring R.**

24. **Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to the field.**

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries **4** *weightage.*

25. **Show that the set of all commutators of a group G generates the smallest normal group C such that G/C is abelian. Determine the commutator subgroup C of D_4 and the factor group $\frac{D4}{C}$.**

26. **State the first isomorphism theorem.**

Let $\phi : Z_{18} \rightarrow Z_{12}$ be the homomorphism where $_4$ (1) =10.

(a) Find the Kernel K of 4.

(b) List the cosets in $_{18}$, showing the elements in each coset.

(c) Find the group $\phi(Z_{18})$.

(d) Give the correspondence between $\frac{Z_{18}}{K}$ and $\phi(Z_{18})$ given by the map W in the first isomorphism theorem.

27. State and prove first Sylow theorem. Show that a normal p-subgroup of finite group is contained in every Sylow *p*- subgroup of G.

28. Determine all group of order 10 up to isomorphism.

(2 x 4 = 8 weightage)