D 72886 (Pages : 3) Name..................................

Reg. **No.**.............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *questions.*
Each question has weightage 1.

1. Let V be a vector space over a field F and $1 \in F$. Prove that $(-1)\,v = -v$ for all $v \in V$.

2. Show that $U = \{(x,0) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

3. Verify whether $\{(1,2,3), (1,3,1)\}$ is a basis for $\mathbb{R}^3$.

4. Give au example of a 2-dimensional subspace of $\mathbb{R}^3$.

5. Find the co-ordinate vector of $(1,2,3) \in \mathbb{R}^3$ with respect to the basis $\{(1,1,0), (1,0,1), (0,1,1)\}$.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (x+1, y+1)$. Verify whether T is a linear transformation.

7. Let $W = \text{span}\{(1,0,0),(1,1,0)\}$. Find a non-zero linear function in W^0.

8. Find the characteristic polynomial of $\begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}$.

9. Find the characteristic values of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

10. Verify whether $W = \{(x,0,0) : x \in \mathbb{R}\}$ is an invariant subspace of $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by :

    $T(x,y,z) = (x+y, y+z, z)$.

Turn over

11. Let W_1 = span {1,2,1} and W_2 = span {(2,1,1), . Verify whether $W_1 + W_2$ is a direct sum.

12. Verify whether $T : R^2 \to R^2$ defined by $T(x, y) = (x + y, 0)$ is a projection.

13. Let V be an inner product space. Prove that II ca $\| = |$ c I II a II for x E **V**.

14. If E is an orthogonal projection of V onto W, prove that a – E_{a} E $W^{\perp}$ for all x E V •

(14 x 1 = 14 weightage)

Part B (Paragraph Type)

Answer any **seven** *questions.*
Each question has weightage 2.

15. Prove that (1,2,3) E R^3 is a linear combination of a = (1,2,1) and β = (1,2,2).

16. Verify whether S = {(x, x ~~E ℝ} is a subspa~~ce of R^2.

17. If W_1, W_2 are subspaces of a vector space V, prove that W_1 n W_2 is a subspace of V.

18. Let V be a vector space of dimension n. Prove that any set of $n + 1$ vectors of V is linearly dependent.

19. Find the matrix of the transformation $T : R^3$ $\mathbf{R}^3$ given by $T(x, y, z) =$ $x+y, x+z, y+z)$ relative to the ordered basis B = {(1,1,0), **(0,1,1), (1,0,1)).**

20. Let {a_1,a_2, , a_n} be a basis of a vector space V and $\{ f_1, f_2,$ $, f_n\}$ be the dual basis of V^*.

Prove that $\mathbf{f} = \sum_{=} f(\alpha_i) f_i$ for each $f \in V^*$.

21. Show that similar matrices have same characteristic polynomial.-

22. Express R^2 as a direct sum of two one-dimensional subspaces.

23. Let T be a linear operator on a vector space V and let V = W_1 10 . . . 8 W_k, where each W_i is invariant under T. Prove that if each W_i is one-dimensional then T is diagonalizable.

24. Verify whether $(x\, I$ y) defined as $(x$ $y) = x_1 + y_1$ is an inner product for :

x = (x_1, = (YDY2) E R^2

(7 x 2 = 14 weightage)

Part C (Essay Type)

Answer any two questions.
Each question has weightage 4.

25. (a) Define linearly independent set in a vector space.

    (b) Let A be an n x n matrix over a field F. Prove that if the row vectors of A form a linearly independent set then A is invertible.

26. Let V be a finite dimensional vector space and $T : V \to V$ be a linear operator. Prove that the following are equivalent :

    (i) T is invertible.

    (ii) T is one-to-one.

    (iii) T is onto.

27. (a) Define the annihilator W^0 of a subspace W of a vector space V.

    (b) Show that if V is finite dimensional then $\dim W + \dim W^0 = \dim V$.

28. (a) Prove that an orthogonal set of non-zero vectors is linearly independent.

    (b) Let W be a subspace of an inner product space V and $\beta \in V$. Show that $\alpha \in W$ is a best approximation to β if and only if $\beta - \alpha \in W^{\perp}$.

(2 x 4 = 8 weightage)