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44042 (Pages : 2) Name..

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SECOND SEMESTER M.Sc. DEGREE EXAMINATION, AUGUST 2013

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA II

ne : Three Hours Maximum : 36 Weightage

Part A

Short answer questions (1-14)

Answer **all** questions. Each question has **one weightage**.

(1) Find all c such that $\mathbb{Z}_5[x]/\langle x \quad + cx + 1)$ is a field.

(2) Define simple extension and give an example of it.

(3) Find a subfied F of R such that it is algebraic of degree **3** over F.

(4) Find a basis of $\mathbb{Q}(\sqrt{\ }, \sqrt{3})$ over Q.

(5) Prove that squaring the circle is impossible.

(6) Is there exist a field of 60 elements? Justify your answer.

(7) Let F, E and K be fields such that $F < E < K$. Prove that $G(K/E) \leq G(K/F)$.

(8) Let F be a field and let a E F. Prove that a is separable over F if and only if $\text{irr}(\alpha, F)$ has all zeros of multiplicity 1.

(9) Prove that a finite extension E of a finite field F is a simple extension of F.

(10) Let σ be the automorphism of $\mathbb{Q}(\pi)$ that maps 7r to —7r. Describe the fixed field *of* σ.

(11) Give an example of two finite normal extensions K_1 and $\mathbf{K_2}$ of the same field F such that K_1 and **K2** are not isomorphic fields but $G(K_1/F) \quad G(K_2/F)$.

(12) How many elements are there in the splitting field of $x^6 - 1$ over $\mathbf{Z_3}$.

(13) Find $1_3(x)$ over **Z2.**

(14) Is splitting field of $x^{\ } - 5$ over Q a solvable **Galois** group? Justify your answer.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven from the following ten questions (15-24).
Each question has weightage 2.

(15) **Prove that a field has no proper nontrivial ideals.**

(16) **Prove that Q($2^1/_2, 2^1/^3$) Q(2^{116}).**

(17) **Prove that the set of algebraic numbers form a field.**

(18) **Prove that there exists a finite field $GF(p^n)$ of p^n elements for every prime power p^n.**

(19) **Show that for a prime p, the splitting field over Q of $x^p - 1$ is of degree p 1 over Q.**

(20) Prove **that the complex zeros of polynomials with real coefficients occur in conjugate pairs.**

(21) **Prove that if E is an algebraic extension of a perfect field F, then E is perfect.**

(22) **Prove that a finite extension of a field of characteristic zero is a simple extension.**

(23) **Prove that the galois group of a finite extension of a finite field is abelian.**

(24) Prove **that the Galois group of the nth cyclotomic extension of Q has $\varphi(n)$ elements.**

(7 x 2 = 14 weighta

Part C

Answer any two from the following four questions (25-28).
Each question has weightage 4.

(25) **Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if RIM is a field.**

(26) **Prove that every field F has an algebraic closure.**

(27) Let **F and E be fields such that $F < E < \overline{F}$. Prove that E is a splitting field** over **F if and only if every automorphism of F leaving F fixed maps E** onto itself and induces an automorphism of **E leaving F fixed.**

(28) **Let F be a field of characteristic 0 and** let a E **F. *If* K is a splitting field of x^n – a over F, then prove that $G(K/F)$ is a solvable group..**

(2 x 4 = 8 weightag