C 83623 **(Pages : 3)** **Name..................................**

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA—II

Time : Three Hours **Maximum : 36 Weightage**

Part A

Short answer questions **(1-14).**
Answer **all** *questions. Each question has 1 weightage.*

1. Is $Q[x]/(x^2-2)$ **a field ? Justify your answer.**

2. **Show that the polynomial x^2+1 is irreducible in** $Z_3[x]$.

3. **Is $\mathbb{C}$ a simple extension over R ? Justify your answer.**

4. **Find [Q ——** $\mathbb{Q}]$.

5. **Let E be a finite extension of degree n over a finite field F. If F has q elements, then prove that E has q^n elements.**

6. **Does there exist a field of 4096 elements ? Justify your answer.**

7. **Prove that a finite extension E of a finite field F is a simple extension of F.**

8. **Find all conjugates of $3+\sqrt{2}$ over** Q.

9. **Find the splitting field of {x^2-2, x^3 − ——over** Q.

10. **Show that** $Q\left(\sqrt[3]{2}\right)$ **has only the identify automorphism.**

11. **Define separable extension of a field. Give a separable extension of** Q.

12. **Let p be a prime, $F=Z_p$ and let $K=GF(p\ \ )$. Find G** (K/F).

13. **Is regular 7-gon constructible ? Justify your answer.**

14. **Prove that the polynomial x^5-1 is solvable by radicals over** Q.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven *from the following ten questions* (15-24).
Each question has weightage 2.

15. Let F be a field. Prove that every ideal in F [x] is a principal ideal.

16. Prove that a finite extension is an algebraic extension.

17. Prove that $Q\left(\sqrt{3}+\sqrt{7}\right)=Q\left(\sqrt{3},\sqrt{7}\right)$.

18. Show that if E is a finite extension of a field F and [E : F] is a prime number, then E is a simple extension of F and E= F (a) for every a E E with a $\notin$ F.

19. Let F be a field and let $f(x)$ be irreducible in F [x]. Prove that all zeros of $f(x)$ in $_F$ have the same multiplicity.

20. Let F be a subfield of a field E. Prove that the set all automorphisms of E leaving F fixed forms a subgroup of the group of all automorphisms of E.

21. Show that if [E : F] = 2, then E is a splitting field over F.

22. Let K be a finite extension of E and E be a finite extension of F. Prove that K is separable over F if and only if K separable over E and E is separable over F.

23. Describe the group of polynomial $(x^3 - 1)$ E Q [x] over Q.

24. Prove that the Galois group of the pth cyclotonic extension of Q for a prime p is cyclic of order $p - 1$.

(7 x 2 = 14 weightage)

Part C

Answer any two from the following four questions (25-28).
Each question has weightage 4.

25. (a) Prove that $x^2 - 3$ is irreducible over $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

(b) Let E be a finite extension field of a field F and let K be a finite extension field of E. Prove that K is a finite extension field of E and

$$[K:F]=[K:E]\,[E:F].$$

26. Let F be a field of characteristic *p*. Prove that the map $6_p : F \to F$ defined by $a_p\,(a) = a$ *is* an automorphism. Also, prove that $F_{\{\sigma_p\}}$ $\mathbb{Z}_p$.

27. Prove that every finite field is perfect.

28. Let F be a field of characteristic zero and let $< E < K < F$, where E is a normal extension of F and K is an extension of F by radicals. Prove that $G\left(E/F\right)$ is a solvable group.

(2 x 4 = 8 weightage)