C 83625 (Pages : 2) Name..............................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CUCSS)

Mathematics

MT 2C 08 TOPOLOGY—I

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type Questions)

Answer **all** *the questions.*
Each question has weightage 1.

1. Write the discrete topology on a set S = {1,2,3}.
2. Give examples of two topologies on a finite set X such that one is weaker than the other.
3. Define co-finite topology.
4. Give an example of an open set, which is not an open interval in the set of real numbers with usual topology. Justify your claim.
5. Write a base for the set of real numbers with usual topology.
6. Define accumulation point of a subset of a topological space. Illustrate using an example.
7. Define quotient map from one topological space to another. Give an example.
8. Define topological property. Write examples of any two topological properties.
9. Give an example of a topological space that is T_o but not T_1.
10. Give an example of a topology on the set of real numbers that is not Hausdorff.
11. Define Tychnoff space.
12. Write an example of a T_4 space.
13. Define second countability and separability in topological space. write a relation between the two.
14. State the Lebesgue covering lemma.

(14 x 1 = 14 weightage)

Part B (Paragraph Type Questions)

Answer any **seven** *questions.*
Each question has weightage 2.

15. Define a metric on the set of real numbers other than the usual metric.
16. Prove that every T_2 space is T_1. Using an example prove that the converse is not true.
17. Lett and ti $_2$ be topologies on a set X and Y be a subset of X. Then prove that if T $_1$ is stronger than $_2$ then t $_1$ /Y is stronger than τ_2 /Y.

Turn over

18. Prove that a subset A of a space X is dense in X if and only if for every non-empty subset B of $A \cap B \quad \phi$.

19. Prove that separable spaces satisfy the countable chain condition.

20. Prove that the product topology is the weak topology determined by the projection functions.

21. Prove that a second countable space is also first countable.

22. Prove that the topological product of any finite number of connected space is connected.

23. Prove that an open subspace of locally connected space is locally connected.

24. Prove that regularity is a hereditary property.

(7 x 2 =14 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question has weightage 4.

25. (a) If a space is second countable then prove that every open cover of it has a countable sub-cover.

(b) Prove that metrisability is a hereditary property.

26. (a) For any subset A of a space X, with usual notations prove that $A = A \cup A'$ •

(b) Prove that closed subset of a compact space is compact.

27. (a) Prove that every continuous real-valued function on a compact space is bounded and attains its extrema.

(b) Prove that every closed and bounded interval is compact.

28. (a) Prove that every regular second countable space is normal.

(b) Let A be a closed subset of a normal space X and suppose $f : A \rightarrow [-1,1]$ is a continuous function. Then prove that there exists a continuous function $F : X \rightarrow [-1,1]$ such that $F(x) = f(x)$ for all $x \in A$.

(2 x 4 = 8 weightage)