C 53090 (Pages : 3) Name..................................

Reg. No..................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS—II

Time : Three Hours Maximum : 36 Weightage

Part **A**

Short answer questions 1-14. Answer **all** *questions.*
Each question has 1 weightage.

1. L et X be a vector space and let dim $X = $ n. Prove that a set E of n vectors spans X if and only if E is independent.

2. Let A E $L($) and let x **E** $\mathbb{R}^n$. Prove that $A'(x) = $ A.

3. Define contraction mapping on a metric space and give an example of it.

4. Let $f =$ $f_2)$ be the mapping of R^2 into R^2 given by

$$f1\,(x, y) = ex\cos y,\ f2\,(x, y) = ex\sin y.$$

Show that the Jacobian of f is not zero at any point of R^2.

5. Find the Lebesgue outer measure of the set $\{1 \pm \frac{\cdot}{2^n} : n = 1, 2, 3, \ldots\}$.

6. Let A and B be measurable sets such that A C B. Prove that $m^*(A) < m^*(B)$.

7. Is the set of irrational numbers in the interval [1,100] measurable? Justify your answer.

8. Prove that constant functions are measurable.

9. Give an example where strict inequality occur in Fatou's lemma.

10. Show that if f is integrable, then so is $|f|$.

11. Let $\{f_n\}$ be a sequence of measurable functions defined on a measurable set E of finite measure. If f_n f $a.e.$, then prove that $\{f_n\}$ converges to f in measure.

12. Show that $D^+[-f(x)] = —D_+ f(x)$.

13. Show that if a $< c < b$, then $T_a^b = T_a^c + T_c^o$.

14. Prove that **sum** of two absolutely continuous functions is continuous. (14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven from the following ten questions (15-24).

Each question has weightage 2.

15. Let Ω be the set of all invertible linear operators on $\mathbb{R}^{n}$. Prove that 1 is an open subset of $L\left(\mathrm{R}^{n}\right)$

16. Let

$$f(x,y)=\begin{cases}0 & \text{if } (x,y)=(0,0)\\ \frac{x^{..}}{x^2+y^2} & \textbf{if } (x,y) \quad (0,0).\end{cases}$$

Prove that $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ exist at every point of $1R^2$.

17. If E_1 and E_2 are measurable, then prove that

$$m(E_1 \cup E_2) \quad m(E_1 \cap E_2)=m(E_1) \quad m(E_2).$$

18. Prove that sum of two measurable functions defined on a same measurable set is measurable.

19. Prove that the characteristic function χ_E is measurable if and only if E is measurable.

20. Let E1, E2, , E_n be disjoint measurable sets and let $co=\sum_{i=1} a_i\chi_{E_i}$. Prove that

$$\int \varphi=\sum_{i=1} a_i m(E_i).$$

21. Let E be a measurable set and let f, g be integrable over E. Prove that $f+g$ is integrable over E and

$$\int_E f+g= \quad _E \quad \int_E g.$$

22. Let f be a function defined by

$$f(x) \quad \begin{cases} O & \text{if } x=0\\ x\sin(\frac{1}{x}) & \textbf{if } x \; L \; 0.\end{cases}$$

Is f differentiable at $x=0$? Justify your answer.

23. If f is of bounded variation on [a, b], then prove that $f'(x)$ exists for almost all x in [a, b].

24. If f is absolutely continuous on [a, bb then prove that f is of bounded variation on $[a,b]$.

(7 x 2 = 14 weightage)

Part C

*Answer any **two** from the following **four** questions (25-28).*
Each question has weightage

25. (a) Let E be an open subset of $\mathbb{R}^n$ and f maps E into m. If f is differentiable at a point x E E, then prove that the partial derivatives $(D_i f_i)(\mathbf{x})$ exist.

(b) If [A] and [B] are n by n matrices, then prove that

$$\text{detail}][B]) = \det[A] \det[B].$$

26. (a) Prove that outer measure of an interval is its length.

(b) Let $\{E_i\}$ be a sequence of measurable sets. Prove that

$$m \left(\cup E_i \right) \leq \quad m(E_i).$$

27. (a) State and prove bounded convergence theorem.

(b) Let $\{f_n\}$ be a sequence of non-negative measurable functions and $f_n(x)$ $f(x)$ almost everywhere on a set E. Prove that

$$_E \mathbf{fn} \quad \lim \int_E f_n.$$

28. Let f be an increasing real valued function on the interval $[a, b]$. Prove that f is differentiable almost everywhere, the derivative f' is measurable and

$$f'(x) \; 5 \; f(b) - (a).$$

(2 x 4 = 8 weightage)