C 44043

(Pages : 3)

Name....

Reg. No

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, AUGUST 2013

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS—II

Time : Three Hours

Maximum : 36 Weightage

Part A

Short answer questions (1-14).

Answer all questions. Each question has one weightage.

(1) Let A E $L(\mathbb{R}^n, \mathbb{R}^m)$. Prove that A is a uniformly continuous mapping of $\mathbb{R}^n$ into $\mathbb{R}^m$.

(2) Prove that the set of all invertible linear operators Ω on $\mathbb{R}$ is an open subset of $L(\mathbb{R}\ )$.

(3) Ler $f: R^2 \to \mathbb{R}^1$ be defined by $f(x, y) = 2x^3 - 3x^2\ 2y^3\ 3y^2$ for (x, y) E R^2. Find the gradient of f at (2, 3).

(4) Define contraction mapping and give an example of it.

(5) Find the outer measure of the set of irrational numbers in the interval [3, 5J.

(6) Prove that finite sets are measurable.

(7) Let f be a continuous real valued function defined on a measurable set E. Prove that f is measurable.

(8) Let $\varphi(x) = \begin{cases} 1 \text{ if } 0 < x < 1 \\ 2 \text{ if } 1 < x < 2 \\ 0 \text{ otherwise.} \end{cases}$

Evaluate φ.

(9) Let f and g be bounded measurable functions defined on a set E of finite measure. If $f < g$ *a.e.*, then show that $\int_E f < g$.

(10) Show that if f is integrable over a measurable set E, then If I is integrable over E.

Turn over

(11) Define convergence in measure. If $\{\quad\}$ is a sequence of measurable functions defined on a mesurable set E of finite measure and f_n f $a.e.$, then prove that fn} converges to f in measure.

(12) If f is of bounded variation on $[a, b]$, then prove that

$$T_a^b = P_a^b + N_a^b$$

(13) Show that $D^+[-f(x)] = -[D_+ f(x)]$.

(14) If f is absolutely continuous on [a, bb then prove that f is of bounded variation on $[a, b]$.

(14 x 1 = 14 weightage)

Part B

Answer any seven from the following ten questions (15-24).
Each question has weightage 2.

(15) Let X be a vector space of dimension n. Prove that a set E of n vectors in X spans X if and only if E is independent.

(16) Let $f(x, y) = \begin{cases} 0 & \text{if } x = 0 \\ \hline & \text{if } (x, y) \quad (0, 0). \end{cases}$

Prove that $D_1 f$ and $D_2 f$ are bounded functions in R^2. Also prove that f is continuous.

(17) Prove that a linear operator A on $\mathbb{R}^n$ is invertible if and only if $\det[A] \quad 0$.

(18) Let C be a collection of subsets of a set X. Prove, that there exists a smallest algebra O containing C.

(19) Show that for any set A and any $\epsilon > 0$, there is an open set O containing A such that $m^*(Q) < m^*(A) \quad \epsilon$.

(20) Prove that if E is a measurable set, then each translte $E \quad$ y of E is also measurable.

(21) Let f and g be real valued measurable functions defined on the same domain. Prove that $f + g$ is measurable

(22) Let $\{f_n\}$ be an increasing sequence of nonnegative measurable functions and let $f = \lim f_n$ $a.e.$. Prove that $\int f = \lim \int f_n$

(23) If f and g are integrable over a measurable set E, then prove that $f + g$ is integrable over E and $\int_E (f \quad g) = \int_E f + \int_E g$.

(24) If f is of bounded variation on $a, b]$, then prove that $f'(x)$ exists for almost all x *in* [a, b].

(7 x 2 = 14 weightage)

Part C

Answer **any two from the following four questions (25-28).**
Each question has weightage **4.**

(25) (i) Prove that a linear **operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X.**

(ii) If *[A]* and *[B]* **are n by n matrices, then prove that**

$$\det([B][A]) = \det[B]\det[A].$$

(26) Prove that the Lebesgue **outer measure of an interval is its length.**

(27) State and prove bounded convergence theorem.

(28) Prove that a function F is an **indefinite integral if and only if it is absolutely continuous.**

(2 **x 4** = 8 weightage)