C 44045 (Pages : 3) Name..................................

Reg. No...............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, AUGUST 2013

(CUCSS)

Mathematics

MT 2C 09—P.D.E. AND INTEGRAL EQUATIONS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has **weightage** *1.*

1. Obtain the partial differential equation satisfied by all surfaces of revolution with z-axis as the axis of revolution.

2. Show that (x ^{2} + (y – + z^2 =1 is a complete integral of z^2 (1 p^2 q^2

3. Determine the domain in which the two equations $f = xp - yq - x = 0$ and $g = x^2 p + q \quad xz = 0$ are compatible.

4. Write the following equation in the Clairant form and solve it :

$$zpq = p^2 (p^2 \quad xq) \quad q^2 (q^2 \quad yp).$$

5. What is an initial strip ?

6. Write the classification of the equation :

$$(n - 1)^2 \, u_{xx} - {}^{2n} \, u_{yy} = ny^{2n-1} \, u$$

7. State the Dirichlet problem and show that the solution of the Dirichlet problem, if it exists, is unique.

8. State the Cauchy problem for the equation $AU_{xx} + BU_{xy} + CU_{yy} = F(x, y, u, u_x, u_y)$ where A, B and C are functions of x and y and give an example.

9. State Harnack's theorem.

10. Differentiate between Fredholm and Voltera integral equations.

11. If y'' = F (x), and y statisfies the initial conditions y (0) = y_0 and y'(0) = A, show that

$$y(x) = \int (x —)F \qquad + \ {}_0' \, x + y_0.$$

Turn over

12. Determine $p(x)$ and $q(x)$ in such a way that the equation $x^2 \frac{d^2 y}{dx^2} \;\; 2x \frac{dy}{dx} + 2y = 0$ is equivalent to the equation $\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q(x)\, y = 0.$

13. Show that the characteristic numbers of a Fredholm equation with a real symmetrical Kernel are all real.

14. Define separable Kernel and give an example of it.

(14 x 1 = 14 weightage)

Part B

Answer **any seven** *questions.*
Each question has weightage 2.

15. Find the general integral of $yzp + xzq = x + y$.

16. Show that the Pfaffian differential equation $(y^2 + yz)\,dx + (xy + z^2)\,dy + (y^2 - xy)\,dz = 0$ is integrable and **find** the corresponding integral.

17. Find the complete integral of $xpq + yq^2 - 1 = 0$.

18. Solve the following equation by Jacobi's method :

$$u_x x^2 - u_y^2 - a u_z^2 = 0.$$

19. Reduce the equation $u_{xx} - 4x^2 u_{yy} = \frac{1}{x} u_x$ into its canonical form.

20. Obtain the D'Alembert's solution which describes the vibrations of an infinite string.

21. Solve the Neumann problem for the upper half plane.

22. Transform the problem $\frac{d^2 y}{dx^2} + y = x$, $y(0) = 1$, $y'(1) = 0$ **to a Fredholm integral equation.**

23. Write a note on Neumann series.

24. Consider the **integral equation** $y(x) = \int_0^1 x\, y(\;)\, d\; + 1$. Show **that the iterative procedure leads to the expression** $y(x) = 1 + x\left(\frac{\lambda}{2} + \frac{\lambda^2}{6} + \frac{\lambda^3}{18} + \cdots\right)$

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. Using the method of characteristics, find the solution of the equation

$z = \frac{1}{2}(p^2 + q^2) + (p - x)(q\ y)$ which passes through the x-axis.

26. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

27. (a) Solve $u_t = \cancel{\quad 0 < x < l, \ t > 0}$

$u(0, t) = u(1, t) = 0$

$u(x, 0) = x(l - x), 0 <— x 5 1.$

(b) Show that the solution of the problem in part (a) is unique.

28. (a) Show that the characteristic values of X for the equation y (x) = $\overset{2n}{}\sin(x + y \quad d\xi$ are

$= \frac{1}{n}$ and ?2 = $-\ 1$ with corresponding characteristic functions of the form

$y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x.$

(b) Obtain the most general solution of the equation y $(x) = \int_0^{2n} \sin(x + y(\xi)\, d \ + x$ under the

assumption that $\lambda \quad 1/\pi.$

(2 x 4 = 8 weightage)