C 44046 **(Pages : 3)** **Name**

Reg. No. . . . .

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, AUGUST 2013

(CUCSS)

Mathematics

MT 2C 10—NUMBER THEORY

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Define Möbius function $\mu(n)$ and show that $\sum_{d/n} t(d) = 0$ if n >

2. Define Mangoldt function $\wedge(n)$ and express $\wedge(n)$ in terms of logarithm.

3. Give an example of a multiplicative function which is not completely multiplicative.

4. Let f be an arithmetical function with $f(1) \neq 0$. Show that $\left(f^{-}\right) = - f' * (f * f^{-1})^{1}$.

5. Derive Selberg identity.

6. Prove that [2x] – [2y] is either 0 or 1.

7. Calculate the highest power of 10 that divides 1000 !.

8. Show that the linear congruence $2x \equiv 3 \pmod 4$ has no solutions.

9. Determine the quadratic residues and non-residues modulo 7.

10. Let (a, m) = 1. Show that $a^{\phi(m)} \equiv 1 \pmod m$.

11. Determine whether -104 is a quadratic residue or non-residue of the prime 997.

12. What is cryptosystem ?

13. Prove that the product of two shift enciphering transformations is also a shift enciphering transformation.

14. How do we send a signature in RSA ?

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question has **weightage** 2.

15. Show that if ——1, then 4)(n) = $\sum_{d/n}$

16. Let f be a multiplicative function. Show that f is completely multiplicative if f $(n) = \mu(n) f(n)$ for all $n \geq 1$.

17. Show that if x 1, then we have $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$.

18. Show that if $x > 2$, then we have $\sum_{n \leq x} \Lambda(n)\left[\frac{x}{n}\right. = x \log x - x + 0\ (\log x)$.

19. Show that for x 2, $\pi(x) = \frac{5(x)}{\log x}$ $\int_2 \frac{(t}{t \log^2 t}\, dt$.

20. Show that for any prime p all the co-efficients of the polynomial $f(x) = (x - 1)(x - 2) \ldots (x - p + 1) - x^{p-1} + 1$ are divisible by p.

21. Show that the Legendre's symbol (n/p) is a completely multiplicative function of n.

22. Show that if p is an odd positive integer then :

(a) $(-1/p) = (-1)^{\left(\frac{p-1}{2}\right)}$

(b) $(2/p) = (-1)^{(p^2-1)/8}$.

23. In the 27-letter alphabet (with blank = 26), use the affine enciphering transformation with key a = 13, b = 9 to encipher the message "HELP ME".

24. Solve the following system of simultaneous congruences :

$x + 4y$ 1 (mod 9).
$5x + 7y$ 1 (mod 9).

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has **weightage** 4.

25. For every integer n 2, prove that $\frac{1}{6}\frac{n}{\log n} < \pi(n) < 6\frac{}{\log n}$.

26. State and prove Lagrange's theorem.

27. State Gauss' lemma. Let m be the number defined in Gauss' lemma. Show that

$$\sum_{t=1}^{\left(\frac{}{2}\right)} \quad p + (n-1)\left(\frac{{}^{2}-1}{8}\right) \pmod 2.$$

28. Find the discrete log of 28 to the base 2 in $\mathbf{F}_{37}$ using the Silver-Pohlig-Hellman algorithm.

(2 x 4 = 8 weightage)