C 63089 (Pages : 3 Name..................................

Reg. No...............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA—II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions (1 –* **14)**
Each question has weightage 1.

1. Verify whether (0, 3} is **an ideal of Z_6.**

2. Verify whether {(0,2n) : n E Z1 **if a prime ideal of Z x Z.**

3. **Verify whether the field Z_5 is an extension of the field Z_3.**

4. Verify whether **Q** **is an algebraic extension of Q.**

5. **Which of the following real numbers is constructible** **a a** $\sqrt[4]{3}$.

6. **Verify** whether $9 : Q\left(\sqrt{2}\right)$ **Q** $\sqrt{3}$ **defined by** $a + b\sqrt{2} \mapsto a + b\sqrt{3}$ **for a, *b* E Q is an isomorphism** of fields.

7. Show that the field **R of reals is not algebrically closed.**

8. Let a be the real cube root of 2. Verify that $Q(\alpha)$ is not a splitting field.

9. Find the order of the group G (Q (a) / (Q) where a is the real cube root of 2.

10. Give an example of an infinite field of characteristic 2.

11. Describe the Galois group G $\left(Q\left(\sqrt{2}\right)/Q\right)$.

12. Let K be a finite field of 8 elements and F = $\mathbf{Z}_2$. Give the order of the Galois group G (K/F).

13. Define the **nth** cyclotomic polynomial.

14. Give an example of a solvable group.

(14 x 1 = 14 wrightage)

Turn over

Part B

Answer any **seven** *questions from the following questions* (15 – 24).
Each question has weightage 2.

15. Find all prime ideals of the ring Z_8.

16. Let R be a ring with identity. Show that the map $\varphi : Z \to R$ defined by $\varphi(n) = n \cdot 1$ is a homomorphism.

17. Let $p(x) = x^2 + 1$ E Q [x]. Let **I** = < $p(x)$> be the ideal generated by $p(x)$. Show that x + I is a zero of $p(x)$ in Q [*x*]/ **I.**

18. Let f (x) = $x^4 - 1$ E Q [x]. Let a $\notin$ Q be a zero of $f(x)$. Find the degree [Q (a) : Q].

19. Let a be a zero of $x^2 + 1$ E Z3 [x]. Find the number of elements in Z_3 (a).

20. Let a be a automorphism of Q Id) with $a(\sqrt{2}) = \sqrt{2}$ and $a(\sqrt{3})$ $\sqrt{3}$. Find the fixed field of a.

21. Let ***f*** (X) E Q *[x]* be irreducible and a, β be zeros **of** ***f (x)*** in $_{Q}$. Let **τ be** an **automorphism** of Q Such that t (a) = β. Let T_x : Q [x] $\to$ Q *[x]* be the natural isomorphism with $\tau_x(x) = x$. Show that $\tau_x(f(x)) = f(x)$.

22. Let F E K and K be a finite normal extension of a filed F. Show that K is a normal extension of E.

23. Let K be a field of a 9 elements and let F = Z_3. Show that a : K K defined by $a(a) = a^3$ for a E K is an **automorphism** of K leaving F fixed.

24. Let K be the splitting field of x^4 + I over Q. Show that G (**K/Q**) is of order 4.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions from the following questions* (25-28).
Each question has weightage 4.

25. Define maximal ideal. Show that if R is a commutative ring with identity and if M is a maximal ideal of R then R/M is a field. Give an example of a commutative ring R with identity, a maximal ideal M of R and describe the field R/M.

26. Let E be an extension of a field F and let a E E. Prove that

    (a) $\varphi_a : F[x]\ E$ defined by $f(x) \mapsto f$ (a) for $f(x)$ e F [x], is a homomorphism.

    (b) If a is algebraic over F then Ker $\varphi_a \neq (0)$.

    (c) If a is transcendental over F then φ_a is one-to-one.

27. Define separable extension. Show that every finite extension of a field of characteristics zero is a separable extension.

28. Define normal extension. Let be a field and $\mathbf{F} \leq \mathbf{E} \leq \mathbf{K} \leq \vec{\mathbf{F}}$. Show that if K is a normal extension of F then K is a normal extension of E.

    Show that G(K/E) is a subgroup of G (K/F).

(2 x 4 = 8 **weightage**)