C 63091 **(Pages : 2)** **Name..............................**

Reg. No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 08—TOPOLOGY—I

Time : Three Hours — Maximum : 36 Weightage

Part A (Short Answer Type Questions)

Answer **all** *the questions.*
Each question has weightage 1.

1. Give an example of an open set in a metric space.
2. Give examples of a discrete topology and an in-discrete topology on a set.
3. Give an example of a closed set in the set of real numbers with usual topology.
4. Distinguish between base and sub-base of a topological space.
5. Define diameter of a set in a metric space. Illustrate using an example.
6. Write an example of a divisible property in a topological space.
7. Distinguish between path connectedness and connectedness in topological spaces.
8. Give an example of a topological space that is T_o but not T_1.
9. Define embedding of a topological space into another.
10. Distinguish between open maps and closed maps in topological spaces.
11. Define mutually separated sets in a topological space. Give example of a pair of mutually separated sets.
12. Define component of a topological space. Give an example.
13. Prove that every regular second countable space is normal.
14. State the Lebesgue covering lemma.

(14 x **1** = **14** weightage)

Part B (Paragraph Type Questions)

Answer any **seven** *questions.*
Each question has weightage 2.

15. Prove that the semi-open interval topology is stronger than the usual topology on the set of real numbers.

Turn over

16. Let $\{x_n\}$ be a sequence in a metric space (X : *d).* Then prove that {x,$_l$} converges toy in X, if for every open set U containing y there exists a positive integer N such that for every integer n N, x_n e U.
17. Prove that second countability is a hereditary property in a topological space.
18. If A and B are any two subsets of a topological space X, prove that $\overline{\text{A u B}}$ = A u B.
19. Prove that the topological product of a finite number of connected spaces is connected.
20. Prove that a set is closed if and only if it contains its boundary.
21. Prove that inverse image of an open set under a continuous function is open.
22. Prove that a compact subset of a Hausdorff space is closed.
23. Prove that every open surjective map is a quotient map.
24. If f X Y is a continuous surjective map, prove that if X is connected then so is Y.

(7 x 2 = 14 weightage)

Part C (Essay Type Questions)

Answer any two *questions.*
Each question has weightage 4.

25. Prove that the usual topology in the Euclidean plane $\mathcal{R}^{-}$ is strictly weaker than the topology induced on it by the lexicographic ordering.
26. Let X be a set, T be a topology on X and S be a family of subsets on X. Then prove that S is a sub-base for T if and only if S generates T.
27. Prove that a subset of the real line is connected if and only if it is an interval.
28. State and prove Urysohn's lemma.

(2 x 4 = 8 weightage)