C 63092

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Name..................................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2014

(CUCSS)

Mathematics

MT 2C 09—PDE AND INTEGRAL EQUATIONS

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 weightage.

1. Determine a partial differential equation satisfied by all surfaces of revolution with z-axis as the axis of revolution.

2. Show that $2z = (ax + y)^2 + b$ is a complete integral of $px + qy - q^2 = 0$.

3. Find the integral of the equation $y\,dx + x\,dy + 2z\,dz = \mathbf{0}$.

4. Determine the region **D** in which the two equations $xp - yq - x = 0$ and $x^{\hat{}}\,p + q - xz = 0$ are compatible.

5. Determine the Monge cone in the case of $p^2 + q^2 = \mathbf{1}$ with vertex (0, 0, 0).

6. Show that if $f(z) = u(x, y) + i\,V(x, y)$ is analytic in $z = x + iy$, then u and v satisfy Laplace's equation in two variables.

7. What is the domain of dependence in the case of a one-dimensional wave equation ?

8. State the Neumann problem.

9. Show that the solution to the Dirichlet problem is stable.

10. State Harnack's theorem.

11. Show that if y" (x) = **F (x) and y satisfies the end conditions y (0) = 0 and y (1) = 0, then**

$$y(x) = \int_0^1 K(x,\xi)F(\xi)\,\mathbf{d}$$, where :

$$K(x,\xi) \quad \begin{cases} \xi(x - 1) \text{ when } < x \\ x(\ -1) \text{ when } \ > x \end{cases}$$

Turn over

12. Show that the characteristic numbers of a Fredholm equation with a real symmetric Kernel are all real.

13. Show that the Kernel K (x,) =1 + 3x has a double characteristic number associated with (-1, 1), with two independent characteristic functions.

14. Determine the resolvent Kernel associated with $K(x,\xi) = \cos(x +$ in (0, 27r), in the form of a power series in A, .

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Find the general integral of the equation (y + 1)p + $(x+1)q = z$.

16. Explain Charpit's method to find a complete integral of the equation $f(x, y, z, p, q) = 0$.

17. Find a complete integral of the equation $p^{\hat{}} x + q^2 y = z$ by Jacobi's method.

18. Find the integral surface for the differential equation :

$z(x_{z_x} - y_{z_y}$ $= y^2 - x^2$ passing through $(2s, s, s)$.

19. Obtain D'Alembert's solution which describe's the vibrations of an infinite string.

20. Reduce the equation $u_{xx} - 4x^{\hat{}} u_{yy} = \frac{?}{x} u_x$ into Canonical form.

21. Solve $u_t = u_{xx}, 0 < x < l, t > 0$

$u(0,t) = u(l,t) = 0$

$u(x,0) = x(l$ x

22. Transform the problem :

$\frac{d^{\hat{}} y}{dx^2} + y$ x, y (0) = 1, y (1) = 0

to a Fredholm integral equation.

23. Solve the **Fredholm** integral equation by iterative method :

$$y(x) = \lambda \int x \xi \, y(\xi) \, d\xi + 1.$$

24. Write a short note on Neumann series.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 weightage.

25. Show that a necessary and sufficient condition that the Pfaffian differential equation :

$$\vec{X} \cdot d\vec{r} = P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$$

be integrable is that $\left(\vec{X} \bullet \text{curl } \vec{X}\right) = 0$.

26. Find the solution of the equation :

$$z = {}^{1}\left(p^{\hat{}} + q^{\hat{}}\right) + (p \quad x)(q \quad y)$$

which passes through the x-axis.

27. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

28. (a) Show that the characteristic values of λ for the equation :

$$y(x) = \int_0^{2n} \sin(x + \xi)\, y(\xi)\, d\xi$$

are $X_i = \frac{1}{\pi}$ and $X2 = -\frac{1}{\pi}$, with corresponding characteristic functions of the form $y_1(x) = \sin x + \cos x$ and $y_2(x) = \sin x - \cos x$

(b) Obtain the most general solution of the equation $y(x) = X \int_0^{2\pi} \sin(x + \xi)\, y()\, d\xi + F(x)$ when $F(x) = x$ and and when $F(x) = 1$, under the assumption that $X \neq \pm \frac{1}{\pi}$

(2 x 4 = 8 weightage)