D 51671 (Pages : 3) Name...

Reg. No

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2013

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Define uniform convergence and give an example of it.

2. Let A be a subset of a topological space X and let $f : A \to \mathbb{R}$ be continuous. Prove that any *two* extensions of f to X agree on $\bar{A}$.

3. Prove that intersection of any families of boxes is a box.

4. Prove that if X_1 is a T_1–space for each $i \in I$ then $\prod_{i \in I} X_i$ is also a T_1–space in the product topology.

5. Let X_i, $i \in I$ be an indexed family of topological spaces and let $X = \prod_{E} X_i$. If $\{x_n\}$ is a sequence in X such that $\{x_n$ converges to $x \in X$, then prove that the sequence $\{\pi_i(x_n)\}$ converges to $\pi_i(x)$ in X_i for each i.

6. State Urysohn's metrization theorem.

7. Let X, Y and Z be topological spaces. If $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then prove that koh and $k' o h'$ are homotopic.

8. Prove that the fundamental group of unit ball in R is trivial.

9. Define covering map and give an example of it.

10. Give an example of a countably compact space.

11. Are compact spaces sequentially compact ? Justify your answer.

12. Let A be a subset of a complete metric space (X ; d) such that A is complete w.r.to. the metric induced on it. Prove that A is closed in X.

Turn over

13. **Prove that finite union of totally bounded sets is totally bounded.**

14. **Prove that $\mathbb{R}$ with usual topology is of second category.**

(14 x 1 = 14 weightage)

Part B

Answer **any seven** *questions.*
Each question has weightage **2.**

15. **Let $\sum_{n=1} M_n$ be a convergent series of non-negative real numbers. Let $\{f_n\}$ be a sequence of real valued functions on a topological space X such that for each $x \in X$ and $n \in N$, $|f_n(x)| \le M_n$. Prove that the series $\sum_{n=1} f_n$ converges uniformly to a real valued function on X.**

16. **Prove that projection functions are open.**

17. **Prove that a product of topological spaces is completely regular if and only if each co-ordinate space is completely regular.**

18. **Let X be a topological space and let $x \in X$. Let e_x be the constant path $e_x : I \to X$ carrying all of I to the point x. If f is a path in X from x_0 to x_1, then prove that $[f]*[e_{x_1}] = [f]$.**

19. **Prove that every continuous, real-valued function on a countably compact space is bounded and attains its extrema.**

20. **Prove that a subspace of a locally compact Hausdorff space is locally compact if and only if it is open in its closure.**

21. **Prove that every compact metric space is complete.**

22. **If a topological space X is regular and locally compact at a point $x \in X$, then prove that x has a local base consisting of compact neighbourhoods.**

23. **Show by an example that total boundedness is not topologically invariant.**

24. **Let (X_1, d_1), (X_2, d_2) be complete metric spaces prove that $X_1 \times X_2$ is complete with respect to the metric :**

$$d((x_1, x_2), (y_1, y_2)) = \max\{d_1(x_1, y_1), d_2(x_2, y_2)\}.$$

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. **Prove that a product of topological spaces is connected if and only if each co-ordinate space is connected.**

26. **Prove that a topological space is completely regular if and only if the family of all continuous real-valued functions on it distinguishes points from closed sets.**

27. **Prove that the map p $\mathbb{R}$ S^{1} given by the equation :**

$$p\,(x) = (\cos 2\pi\, x, \sin 2n\, x)$$

is a covering map.

28. **Prove that any continuous function from a Tychnoff space into a compact, Hausdorff space can be extended continuously over the stone Cech compactification of the domain.**

(2 x 4 = 8 weightag