D 71327 **(Pages : 3)** **Name..**

Reg. No......................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Show that if a linear transformation has ∞ for its only fixed point, then it is a translation.
2. State the symmetry principle.
3. Compute the cross ratio :

   $(i, 0, -1, \text{co})$.

4. Find the image of the hyperbola $\{z = x + iy : x^2 - y^2 = 1\}$ under the map $f(z) = z^2$.
5. State the Cauchy's Integral Formula.
6. Compute $\int_{r} \text{- - - -}\, dz$, where $r(t) = e^{\cdot\cdot}, 0 < t < 2\pi$
7. What is the nature of the singularity of e^z at $z = \infty$?
8. State general form of Cauchy's theorem.
9. Find the nature of the singularity of the function $\frac{1}{\sin^2 z}$ at $z = 0$.
10. Let u be a real valued piecewise continuous function on $[0, 2\pi]$. Define the Poisson integral of u.
11. Find the roots of the equation :

    $z^4 - 6z + 3 = 0$.

    in the annulus {z :1 <

Turn over

12. Obtain the power series expansion of $\frac{1}{-}$ about $z = 1$ in the disk $_{z}$ z -1 I < 1} .

13. Prove that the sum of the residues of an elliptic function is zero.

14. Show that there does not exist an elliptic function with a single simple pole.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Prove that a linear transformation carries circles into circle.

16. Describe the mapping properties of $W = e^z$.

17. State and prove Schwarz's lemma.

18. Let r be a closed rectifiable curve. For any point 'a' not on r define n $(r,$ a). Show that n (r, a) is always an integer.

19. Show that if $f(z)$ is analytic in a region Ω and statistics the inequality I f $(z) - 1 \,|< 1$ on Ω, then :

$$\int_r \frac{f(z)}{f(z)} dz = 0,$$ for every closed curve r in Ω.

20. Obtain the Laurent series expansion of $\frac{1}{z(z-1)}$ in :

(i) $0 < |\, z \,| < 1$. (ii) I $z \,| > 1$.

21. State and prove the Residue theorem.

22. If $f(z)$ is analytic in a region Ω and has no zeros in Ω, prove that log $f(z)\, I$ is harmonic in c.

23. Suppose f has an isolated singularity at $z = \text{a}$. **If** $\lim\limits_{z \to a} (z - \text{a}) \quad (z) = 0$, show that $z = \text{a}$ is a removable singularity.

24. Show that any even elliptic functions with periods w_1 and w_2 can be expressed in the form :

$$C \frac{- \quad p(z) - p(a_k)}{k = 1 \; p(z) - p(b_k)}$$

where C is a constant.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 weightage.

25. State and prove Cauchy's theorem for a rectangle.

26. Using Residue theorem, evaluate the integral $\int_{0}^{-} \frac{\;}{a+\cos\theta}$ where $a > 1$.

27. Derive the formula for the Weierstrass elliptic function in the form :

$$p(z) = \frac{1}{z} + \sum_{w \neq 0} \left(\frac{1}{(z-w)} \right.$$

28. Derive the Poisson integral formula for harmonic functions.

(2 x 4 = 8 weightage)