D 71330 (Pages : 3) Name..............................

Reg. No..............................

# THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 3C 14—LINEAR PROGRAMMING AND ITS APPLICATIONS

Time : Three Hours Maximum : 36 Weightage

## Part A (Short Answer Type)

*Answer* **all** *the questions.*
*Each question carries weightage 1.*

1. Is union of two convex sets a convex set ? Justify your answer.
2. Prove that a hyperplane is a convex set.
3. Is the function $f(x) = x, x \in R$, a convex function ? Justify your answer.
4. Find the Hessian of $f(X)$ where $f(X) = x_1^3 + 2x_2^3 + 3x_1 x_2 x_3 + x_3^2$.
5. Define the dual of a linear programming problem. Give an example.
6. What is meant by loops in a transportation array ?
7. Describe the concept of degeneracy in transportation problem.
8. Describe the generalized transportation problem.
9. Describe the $0 - 1$ variable problems in integer programming.
10. Define artificial variables. Describe the uses of artificial variables in solving linear programming problems.
11. Describe the fixed charge problem in integer programming.
12. Describe the concept of primal and dual problems in optimization theory.
13. Describe the notion of dominance in game theory.
14. Describe matrix games.

Turn over

## Part B (Paragraph Type)

*Answer any* **seven** *questions.*
*Each question carries weightage* **2.**

15. Obtain a necessary and sufficient condition for a differentiable function in a convex domain to be convex.

16. Find the convex hull of the points (1, 0, 0), (0, 1,0) and (0,0,1) in $E_3$.

17. Use the method of Lagrange multipliers to find the maxima and minima of $x_2^2 - (x_1 + 1)^2$ subject to $x1^2 + X2^2 < 1$.

18. Show that the function

$f(x) = x_1^{\frown} + 4x_2^2 + 4x_3^2 + 4x_1 x_2 + 4x_1x_3 + 16x_2x_3$ has a saddle point at the origin.

19. Define a polytope. Prove that a point $X_\upsilon$, of a polytope is a vertex if and only if $X_\upsilon$, is the only member of the intersection set of all the generating hyperplanes containing it.

20. Describe unbalanced transportation problem.

21. Prove that the transportation problem has a triangular basis.

22. Describe the rectangular game as a Linear programming problem.

23. Write the general form of an integer linear programming problem.

24. State and prove the mini max theorem in theory of games.

## Part C (Essay Type)

*Answer any* **two** *questions.*
*Each question carries weightage* **4.**

25. Solve the following problem using simplex method :

Maximize $5x_1 - 3x_2 + 4x_3$

subject to

$$
\begin{array}{rr}
x_i - x_2 & 1 \\
-3x_1 + 2x_2 + 2x_3 & 1 \\
4x_1 - x_3 \; = & 1 \\
0, X2 & 0
\end{array}
$$

$x_3$ unrestricted in sign.

26. Solve the transportation problem for minimum cost with cost coefficients, demands and supplies as given in the following table. Obtain three optimal solutions.

| | D1 | D2 | D3 | | |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | -2 | 3 | 70 |
| $O_2$ | 2 | 4 | 0 | 1 | 38 |
| O3 | 1 | 2 | -2 | 5 | 32 |
| | 40 | 28 | 30 | 42 | |

27. Solve the following integer linear programming problem :

Maximize 4) (X) $= 3x_1 + 4x_2$

subject to $2x1 + 4x_2 < 13$

$-2x_1 + x_2 \quad 2$

$2x_1 + 2x_2 \quad 1$

$6x_1 - 4x_2 \leq 15$

$x_i, X2 \quad 0$

$x_1$ and $x_2$ are integers.

28. Solve the game where the payoff matrix is :

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$