D 91596 **(Pages : 3)** **Name.....................................**

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that, if a product is non-empty, then each projection is onto.

2. Let be the product topology on the set 1 1 X, where $\{(X_i, \tau_i) : i \text{ E I}\}$ is an indexed collection of
   1 E1

   topological spaces. Prove that the family of all subsets of the form (V,) for V, E t,, $i \in$ I is a subbase for τ.

3. Prove that a topological product is T_o if each coordinate space is T_o.

4. Let $X = \prod_{i \in I}$ X where each X is a topological space and let x E X. Suppose $\{x_n\}$ is a sequence in X with the property that for each i E I, the sequence $\{\pi_i$ x,, converges to π_i (x) in X . Prove that $\{x_n\}$ converges to x in X.

5. Prove that the evaluation function of a family of functions is one-to-one if and only if that family distinguishes points.

6. Give an example of a second countable Hausdorff space which is not metrizable.

7. Prove that in a simply connected space, any two points having the same initial and final points are path homotopic.

8. Show that if $p : E \to B$ is a covering map, then p is an open map.

9. Prove that the map $p :$ $\to$ S' given by $p(z) = z^2$ is a covering map.

10. Prove that a continuous real-valued function on a countably compact space is bounded and attains its extrema.

11. Prove that a first countable countably compact space is sequentially compact.

Turn over

12. Let X be Hausdorff and locally compact at a point x in X. Prove that the family of compact neighbourhoods of x is a local base at x.

13. Let A be a subset of a metric space (X, d) such that A is complete with respect to the metric induced on it. Prove that A is closed in X.

14. Prove that the set of irrationals is of second category in the real line R with the usual topology.

(14 x 1 = 14 marks)

Part B

Answer any seven *questions.*
Each question has weightage 2.

15. Prove that any continuous real-valued function on a closed subset of a normal space can be extended continuously to the whole space.

16. Let $\{X \ : i \in I\}$ be an indexed family of sets and let $X = \prod_{i \in I} X$, Prove that a subset of X is a box if and only if it is the intersection of a family of walls.

17. Let S be a subbase for a topological space X. Prove that X is completely regular if and only if for each V $\in$ S and for each x $\in$ V, there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all y V.

18. Let $\{f_i : X \to Y \mid i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Prove that the corresponding evaluation function $e : X \to \prod_{i \in I} Y$, is open when regarded as a function from X onto e (X).

19. Prove that path homotopy $(\simeq p)$ is an equivalence relation on the set of paths in a topological space X.

20. Let $P : E \to B$ be a covering map and let $p(e_0) = b_0$. Prove that any path $f : [0, 1] \to B$ beginning at b_0 has a lifting to a path f in E beginning at e_0.

21. Prove that a subset of Euclidean space is compact if and only if it is closed and bounded.

22. Among all Hausdorff compactifications of a Tychonoff space, prove that the stone-cech compactification is the largest compactification upto a topological equivalence.

23. Prove that every compact metric space is complete.

24. Prove that the function $e : \hat{X} \times \hat{X} \to R$ defined by $e([\hat{x}], [\hat{y}]) = \lim d$ (x,,, (where $\hat{x} = \{x\}$ and $\hat{y} =$ y,,}) is a metric on X, where (X, d) is a metric space.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. Prove that a product of spaces is connected if and only if each co-ordinate space is connected.
26. State and prove Urysohn's metrization theorem.
27. Prove that the fundamental group of the circle is infinite cyclic.
28. Prove that a topological space is compact if and only if there exists a closed sub-base C for X such that every family of C having the finite intersection property has a non-empty intersection.

(2 x 4 = 8 weightage)