D 31328 **(Pages : 3)** **Name.**

Reg. No.................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

(CUCSS)

Mathematics

MT 3C —LINEAR PROGRAMMING AND ITS APPLICATIONS

(2010 Admissions)

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer* all *questions.
Each question has weightage 1.

1. **Prove that the set of all convex linear combinations of points of a set is a convex set.**
2. **Define a closed half-space and a polytope.**
3. **Find the convex hull of the set S** $= \{(0, 0, 0), (1, 0, 0), (0,1, 0), (0, 0;)\}$ **in** E_3.
4. **Find** $\nabla f(X)$ **and** $H(X)$ **for** $f(X) = \quad + 2x2 + 3x_1x_4x_5 + x_3^2$.
5. **Define the basic feasible solution in an LP problem.**
6. **Explain briefly the term 'Artificial variables'.**
7. **Write the dual of the LP problem.**

Maximize $+ 6x_2 + 4x_3 \; 6x_4$

subject to $2x, +3x_2 +17x_3 +80x_4 \; 48$

$8x, + 4x_2 + 4x + 4x_4 = 21$

$x_1, x_3 > 0$

x_4 unrestricted in sign.

8. What do you mean by loops in a transportation array ?
9. What is degeneracy in transportation problem ?
10. Explain the difference between transportation problem and assignment problem.
11. What is integer linear programming ?
12. Describe a two-person zero-sum game.

Turn over

13. Solve the game whose pay-off matrix is given by $\begin{vmatrix} 1 & 7 & 3 & 4 \\ 5 & 6 & 4 & 5 \\ 7 & 2 & 0 & 3 \end{vmatrix}$

14. Explain the notion of dominance in rectangular games.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Define vertex of a convex set. Give an example of a convex set in which all boundary points are vertices.

16. Find the point in the plane $x_1 + 2x_2 + 3x_3 = 1$ in E_3 which is nearest to the point $(-1, 0, 1)$.

17. Prove that the linear function $F(X) = CX$, $X \in E_n$ is both convex and concave.

18. Prove that a vertex of the set S_F of feasible solutions is a basic feasible solution.

19. Explain briefly the simplex procedure to solve the linear programming problem.

20. Obtain an initial basic feasible solution to the transportation problem

	D_1	D_2	D_3	D_4	
O_1	1	2	–2	3	70
O_2	2	4	0	1	38
O_3			–2	5	32
	40	28	30	42	

21. Prove that the optimum value of the primal of an LP problem if it exists is equal to the optimum value of its dual.

22. Describe the Caterer problem in Operations Research.

23. Explain whether an integer programming problem can be solved by rounding off the corresponding simplex solution.

24. Write both the primal and the dual LP problems corresponding to the rectangular game with the pay-off matrix.

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. **Prove that every point of the convex hull of a set S c E. can be expressed as a convex linear combination of atmost + 1) points of S.**

26. **Solve using simplex method**

Maximize $Z = 5x_1 + 3x_2 + x_3$

subject to $2x_1 + x_2 + x_3 = 3$

$x_1 + 2x_3 = 4$

$x_1, x_2, x_3 \quad 0.$

27. **Describe the cutting plane method for solving an integer linear programming problem.**

28. **Solve graphically the game whose** pay-off matrix **is given by** $\begin{pmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{pmatrix}$.

(2 x 4 = 8 weightage)