D 31326 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

(CUCSS)

Mathematics

MT 3C 12—FUNCTIONAL ANALYSIS—I

(2010 admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *question&*
Each question carries 1 **weightage.**

1. Prove or disprove : A sequence (x_n) in the metric space $1^7,1\ 5\_p\ 5\_\ a,$ converges to x in if $x_n(j) \to x(j)$ for each $j = 1, 2, 3, \ldots$
2. Give an example of a bounded sequence in a metric space which is not cauchy.
3. State Minkowski's inequality for measurable functions on a measurable subset of R.
4. Define nth Dirichlet Kernal D_n and show that $\int D_n(t)\,dt = 2\pi$
5. Let Y be a subspace of normed space X. Show that is a normed space.
6. Define inner product space.
7. State Gram-Schmidt orthonormalization theorem.
8. Let $(\ _n)$ be a sequence in a Hilbert space H. Show that if $\sum_{n=1} \| x_n \| < \infty$, then $\sum_n x_n$ converges in H.
9. Let X be an inner product space. Let $E \subset X$ and $x \in E$. Show that there exists a best approximation from E to x if $x \in E$.
10. Let X be a normed space, $f \in X'$ and f 0. Let $a \in X$ with $f(a) = 1$ $r > 0$. Show that $U(a,r) \cap Z(f) \neq 0$ iff $\| f \|\ 1$.
11. Show that C_{oo} is not closed in r
12. What is the geometrical interpretation of the uniform boundedness principle ?

Turn over

13. Let X be a normed space over K and $x \in X$. Define j_x K by $f_x(f) = f(x)$ for $f \in$. Show that $j_x \in X''$ and II ix 11'11 x11.

14. Let X be a normed space and (x_n) be a sequence in X such that $(f(x_n))$ converges in K for every $f \in X$ • Show that the sequence (x_n) is bounded.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Show that a non-empty subset of a separable metric space is separable in the induced metric.
16. State and prove Riemann–Lebesgue lemma.
17. Prove that the three norms $\| \|_1$, $\| \|_2$ and II $\|$ are equivalent.
18. Let X and Y be normed spaces and Z be a closed subspace of X. Show that if F 6 BL$(X/Z$, Y) and we let $F(x) =$ + Z) for $x \in X$, then F c BL (X, Y) d F H
19. Let <, > be an inner product on a linear space X and T x X be a linear one-to-one map. Let : $< x, y >_T =< T(x), T(y) >$ for $x, y \in X$.

    Show that $<, >_T$ is an inner product on X.
20. State and prove Bessel's inequality.
21. Let $X = C([-1,1]), x(t) = 1 - t$, $x_0(t) =$ and $x(t) = \cos pct$ for t —1,1}. Show that the best approximation from span $\{x_0, x_1\}$ to x is $4x_1$
22. Let Y be a subspace of a normed space X and $a \in X$ but $a\ y$. Show that there is some $f \in$ such that $f/_Y = 0$, $f(a) = \text{dist}(a, \bar{Y})$ and II f 11=1.
23. Show that a normed space X in a Banach space iff every absolutely summmable series of elements in X is summable in X.
24. Let X be a normed space and E be a subset of X. Show that E is bounded in X iff f (E) is bounded in K for every $f \in X$.

= 14 weightage)

Part C

Answer any two questions.
Each question carries* 4 *weightage.

25. Show that for 1 $p <$ co, the metric space l' is separable, but r is not separable.

26. Show that every finite dimensional subspace of a normed space X is closed in X.

27. Show that a non-zero Hilbert space H is separable if H has a countable orthonormal basis.

28. Let. X be a normed space. Show that for every subspace Y of X and every $g \, \varepsilon \, \mathbf{y}'$, there is a unique Hahn–Banach extension of g to X if X is strictly convex.

2 X 4 = 8 weighatge)