**D 31327** **(Pages : 3)** **Name**

**Reg. No. . . . . . . . . . . . . . . .**

# THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

**(CUCSS)**

**Mathematics**

**MT 3C 13—TOPOLOGY—II**

**(2010 Admissions)**

**Time : Three Hours** **Maximum : 36 Weightage**

## Part A

*Answer* **all** *questions.*
*Each, question has weightage 1.*

1. Prove that the intersection of any family of boxes is a box.
2. Prove that the projection functions are open.
3. Prove that if; is a $T_2$-Space far each $i \in I$, then $\pi_{i \in I} X_i$ is also a $T_2$-space in the product topology.
4. Explain countably productive topological property.
5. Let $\{Y_i : i \in I\}$ be a family of sets, X a set and for each $i \in I$, $Y_i$ a function. Prove that the evaluation function is the only function from X into $\pi Y_i$ whose composition with the projection $\pi_i$ $Y_i$ equals $f_i$ for all $i \in I$.
6. State a necessary and sufficient condition for embedding a topological space in the Hilbert cube.
7. Define a simply connected space and *give* an example.
8. Show that if $P : E$ B is a covering map, then $p$ is an open map.
9. Let a covering map $P : R$ S' be given by $p(x) = (\cos 2\pi x, \sin 2n x)$. Find a lift of the path : [0,1] S' defined by $f(x) = (\cos \pi x, \sin ax)$.
10. Prove that a continuous real-valued function on a countably compact space is bounded and attains its extrema.
11. Let $\mathcal{B}$ be a base for a topological space X such that every cover of X by members of *3* has a finite subcover. Prove that X is compact.

**Turn over**

12. Prove that every locally compact Hausdorff space is regular.

13. Give an example of a metric space which is complete but not compact.

14. Define Completion of a metric space (X, *d*).

**(14 x 1 = 14 weightage)**

## Part B

*Answer any* **seven** *questions.*
*Each question has* **weightage** *2.*

15. Let A be a closed subset of a normal space X and suppose $f : A \to (-1,1)$ is continuous. Prove that there exists a- continuous function F: X (-1,1) such that F $(x) = f(x)$ for all $x$ c A.

16. Let $\{(x_i, \tau_i) : i$ E I} be an indexed collection of topological spaces and let τ be the product topology on the set $_{i \in I}$ $i$ Prove that $\{$ $(V_1) : V_1$ E $\tau_i, i$ E I $\}$ is a sub-base for T.

17. Let (X, *d*) be a metric space and let X. be any positive real number. Prove that there exists a metric *e* on X such that *e* $(x, y)$ λ for all $x$, y E X.

18. Prove that a topological space is a Tychonoff space iff it is embeddable into a cube.

19. Prove that path homotopy ——*p*) is an equivalence relation on the set of paths in a topological space X.

20. If $h : (X, x_0) \to (Y, y_0)$ is a homeomorphism of X with Y, prove that $h_*$ *is* an isomorphism of $\bar{}_1$ (X, x0) with $\pi_1$ (Y, $y_0$).

21. Prove that sequential compactness is preserved under continuous functions.

22. Let X be a Tychonoff space, (e, (x)) its Stone-Cech compactification and suppose $f : X \to [0, 1]$ is continuous. Prove that there exists-a map $g : \beta$ (X) → [0,11 such that goe = *f*.

23. Let A be a subset of a metric space (X, *d*) such that A is complete with respect to the metric induced on it. Prove that A is closed in X.

24. Prove that the range of the embedding $h : (X, d)$ $(\hat{X}, e)$ is a dense subset of $\hat{X}$ in the metric topology induced by e.

**(7 x 2 = 14 weightage)**

## Part C

*Answer any two questions.*
*Each question has weightage 4.*

25. Prove that metrisability is a countably productive property.

26. Prove that a topological space is embeddable in the Hilbert cube iff it is second countable and $T_3$.

27. Let $p : E \rightarrow B$ be a covering map and let $p\,(e_0) = b_{\upsilon}$. Let the map F:1 x I $\rightarrow$ B be continuous with F (0, 0) = $b_{\upsilon}$. Prove that there is a lifting of F to a continuous map such that

    (0, 0) = $e_{\upsilon}$.

28. State and prove Alexander sub-base theorem.

(2 x 4 = 8 weightage)