D 31618 **(Pages : 2)** **Name**

Reg. No..................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

(Non-CUCSS)

Mathematics

Paper XIII—TOPOLOGY—II

(2002 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 4 *marks.*

I. (a) Let A be a subset of the topological space X and let $f : A \to \mathbb{R}$ be continuous. Prove that any two extensions of f to X agree on A.

(b) Prove that the intersection of any family of filters on a set is again a filter on that set.

(c) Prove that first countable, countably compact space is sequentially compact.

(d) Prove that a subspace of a locally compact space Hausdorff space is locally compact if and only if it is open in its closure.

(4 X 4 = 16 marks)

Part B

Answer any **four** *questions without omitting any unit.*
Each question carries **16** *marks.*

Unit I

II. (a) Let X be a topological space having the property that for each closed subset A of X, every continuous real valued function on A has a continuous extension to X. Prove that Xis normal.

(b) If a product space is non-empty, then prove that each co-ordinate space is embeddable in it.

III. (a) Let $\{X_i : i \, E \, I\}$ be an indexed family of sets and let $X = \prod_{i \in I} X_i$. Prove that a subset of a topological space X is a box if and only if it is the intersection of a family of walls.

(b) Prove that a topological product is T_2 if and only if each co-ordinate space is T_2.

IV. (a) Let T be the product topology on the set $\prod_i V$ where $\{(X_i, T_i) : i \in I\}$ is an indexed collection of topological spaces. Prove that the family of all subsets of the form $\pi_i^{\cdot}(V_i)$ for $V_i \, E \, T_i$, $i \, E \, I$ is a subbase for T.

Turn over

(b) Prove that a product of topological spaces is path connected if and only if each co-ordinate space is pathconnected.

Unit II

V. (a) Prove that a topological space is a Tychnoff space if and only if it is embeddable into a cube.

(b) Prove that a space is embeddable in the Hilbert cube if and only if it is second countable and T_s.

VI. (a) Let $S \longrightarrow X$ be a net in a topological space X and let x E X. Prove that x is a cluster point of S if and only if there exists a subnet of S which converges to x in X.

(b) Prove that a topological space is compact if and only if every family of closed subsets of it, which has the finite intersection property, has a non-empty intersection.

VII. (a) Prove that a topological space is Hausdorff if and only if no filter can converge to more than one point in it.

(b) Prove that a topological space is compact if and only if every ultra filter in it is convergent.

Unit III

VIII. (a) Prove that sequential compactness is a countably productive property.

(b) If a topological space X is Hausdorff and locally compact at point x E X, then prove that the family of compact neighbourhoods of x is a local base at x.

IX. (a) Prove that a topological space X is compact if and only if there exists a closed sub-base C for X such that every subfamily of C having the finite intersection property has a non-empty intersection.

(b) Prove that every compact metric space is complete.

X. (a) Let A be a subset of a metric space $(X ; d)$. Prove that A is totally bounded with respect to d if and only if for every $\in > 0$, A can be a covered by finitely many open balls with centres in A and of radii less than E each.

(b) Prove that every metric space can be isometrically embedded as a dense subspace of a complete metric space.

(4 x 16 = 64 marks)