D 51669 **(Pages : 3)** **Name**

Reg. No.....................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2013

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Find the fixed points of the linear transformation w = $\frac{2z}{3z-1}$

2. Find the points at which the function tan z is not analytic.

3. State the symmetry principle.

4. If $z = x + iy$, prove that $|e^z = ex.$

5. Compute $\int ydz$ where r is the directed line segment from 0 to i.

6. Let n be a positive integer. Prove that $\int (z-a)^n\, dz = 0$ for any closed curve r.

7. Let the curve r lie inside of a circle. Prove that the index n $(r, a) = 0$ for all points a outside of the same circle.

8. Determine the nature of the singularity of the function $\frac{\sin z}{}$ ~~at $z = 0$. Justify~~ your answer.

9. Find the residue of the function $f(z) = \frac{z^2 - 2}{(z-2)^2}$ at $z = 2$.

10. Define : Simply connected region. Give an example of a simply connected region.

11. Prove : the argument 0 is harmonic wherever it can be defined.

Turn over

12. **Find a harmonic conjugate of the function** $e^x \cos y$.

13. **Find the Taylor series expansion of the function** $\frac{1}{z^2}$ **at** $z = 1$.

14. **Prove that an elliptic function without poles in constant.**

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries **2** *weightage.*

15. **Let Q be the region C – {z : z e., the complement of the negative real axis. Define a continuous function** $f : \Omega \to \mathbb{C}$ **satisfying** $f(z) = z^2$ **for all** $z \in \Omega$ **and f (1) = 1. Show that** f **is analytic in Q.**

16. **Define : Linear fractional transformation prove that a linear fractional transformation is a topological mapping of the extended complex plane into circles.**

17. **Prove that the cross-ratio** (z_1, z_2, z_3, z_4) **is real if and only if the four points lie on a circle or a straight line.**

18. **Let** f **be a continuous complex valued function defined on the closed interval [a, b]. Prove that**

$$\left| \int_a f(t)\, dt \right| \quad \int_a f(01\, dt.$$

19. **State and prove Morera's theorem.**

20. **Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.**

21. **How many roots does the equation** $z^7 - 2z^5 + 7z^3 - z + 1 = 0$ **have in the disc** $|z| < 1$.

22. **State and prove Hurwitz's theorem.**

23. **Find the Laurent series expansion of the function** $I(z) - \frac{1}{(z-1)(z-2)}$ **in the regions** $0 < |z \quad {}^{1}| < 1$ **and** $1 < |z - 2| < \infty$.

24. **Derive Legendre's relation** :X11 W2 $= 2\pi i$.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Let the function $f(z)$ be analytic on the rectangle R defined by the inequalities

Then prove that $\int_{\partial R} f(z)\,dz = 0$.

26. Let the function f be analytic in a region Q and let a E S2 . Suppose that f(a) and all its derivatives $f^{(n)}$ (a) vanish. Prove that f is identically zero in Ω.

27. Discuss the evaluation of integrals of the type $\int R(x)\, e^{ix}\, dx$ using the theory of residues.

28. Prove that the Weirestrass eliptic function satisfies the differential equation of the form

$$(z))^2 = 4(\wp(z)\;^3 - g_2\, \text{O}\,(z) - g3.$$

(2 x 4 = 8 weightage)