C 43673 (Pages e 2) Name

Reg. No.....................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JULY 2013

(Non-CUCSS)

Mathematics

FLUID DYNAMICS

Time : Three Hours Maximum : 80 Marks

Answer **all** *the questions from Part* A *and any* four *questions from Part* B *without omitting any unit.*

Part A

Each question carries 4 *marks.*

1. Show that a vortex filament cannot terminate at a point within the fluid.
2. Show that in a simply connected region the only possible irrotational motion is acyclic.
3. What is cavitation ? Explain.
4. Discuss the image of a doublet in a plane.

(4 x 4 = 16 marks)

Part **B**

Each question carries 16 *marks.*

UNIT I

I, (a) Establish the equation of continuity for an incompressible fluid in the form

$$\frac{au}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \quad 0.$$

(b) Determine the condition that u $= ax + by,\ v = cx + dy$ may give the velocity components of a possible incompressible fluid motion in two dimension.

II. (a) Derive the equation of motion of an inviscid fluid.

(b) State and prove Kelvin's minimum energy theorem.

III. (a) Show that in irrotational motion the curves of constant velocity potential cut the streamlines orthogonally.

(b) In two-dimensional irrotational motion, prove that, if the speed is everywhere the same, the streamlines are straight.

Turn over

UNIT II

IV. (a) Describe the streaming motion past a circular cylinder.

(b) Prove, or verify, that the velocity potential $\phi = u\left(r + \frac{a^2}{r}\right)\cos\theta$ represents a streaming motion past a fixed circular cylinder.

V. (a) Show that the Joukowski transformation maps concentric circles with centre at the origin in the z-plane into confocal ellipses in the z-plane.

(b) State and prove Blasius's theorem.

VI. (a) Discuss the geometrical construction for Joukowski aerofoils.

(b) State and prove the theorem of Kutta and Joukowski.

UNIT III

VII. (a) Suppose that there is a source of strength m at $A(a, 0)$, and a sink of strength m at B(— a, 0) and a uniform stream U parallel to the real axis. Determine the stream function.

(b) Discuss the effect on a wall of a source parallel to the wall.

VIII. (a) If we map the z-plane on the -plane by a conformal transformation $= f(z)$, then show that a source in the z-plane will transform into a source at the corresponding point of the -plane.

(b) Prove that in conformal transformation a doublet will transform into a doublet, but that the strength will differ.

IX. (a) A and B are a simple source and sink of strengths μ and respectively, in an infinite liquid. Show that the equation of the streamlines is $\mu\cos\theta - \mu'\cos\theta' =$ constant, where θ, θ' are the angles which AP, BP make with AB, P being any point.

(b) Verify that $= \frac{A}{2}\cos\theta + Br^2\sin^2\theta$ is a possible form of Stoke's stream function, and find the corresponding velocity potential.

(4 x 1.6 = marks)