C 63398 (Pages : 3) Name

Reg. No.....................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2014

(Non-CUCSS)

Mathematics

Paper XVI—FUNCTIONAL ANALYSIS—II

Time : Three Hours Maximum : 80 Marks

Part A

Answer all *questions.*
Each question carries 4 *marks.*

1. Let X be a normed space. Show that BL (X) is closed with respect to composition of functions and that the composition is continuous.
2. Give an example of a normed space which is reflexive but not strictly convex.
3. Let X be an inner product space and E c X be convex. Show that there exists at most *one* best approximation from E to any x E X.
4. Let H be a Hilbert space and A s BL (H) be self-adjoint. Show that A^2 0 and $A \leq \|A\|$ I.

(4 x 4 = 16 marks)

Part B

Answer any four *questions without omitting any unit.*
Each question carries 16 *marks.*

UNIT I

I. (a) Let X be a normed space and A c BL (H) be of finite rank. Show that :

$$\sigma_e(A) = \sigma_a(A) = \sigma(A).$$

(b) Let X be a Banach space over K and A c BL (X) . Show that $\sigma(A)$ is a compact subset of K .

II. (a) Let X be a normed space. Show that if X' is separable, then so is X. Is the converse True ? Justify your answer.

Turn over

(b) Let 1 p ∞ and $\frac{1}{p}$ $\frac{1}{q}$ = 1. Show that the dual of K" with the norm is linearly isometric to K" with the norm .

III. (a) Show that every closed subspace of a reflexive normed space is reflexive.

(b) Let Y be a closed subspace of a normed space X. Show that X is reflexive if Y and $X/_{Y}$ are reflexive.

UNIT II

IV. (a) Let X be a normed space and Y be a Banach space. Show that CL (X, Y) is a closed subspace of BL (X, Y) .

(b) Is it true that every continuous linear map on a normed space is compact ? Justify your answer.

V. (a) Let X be a normed space and A ε CL (X). Show that every non-zero spectral value of A is an eigenvalue of A.

(b) Let X be a normed space and A c CL (X). Show that every eigen space of A corresponding to a non-zero eigen value of A is finite dimensional.

VI. (a) State and prove Riesz representation theorem.

(b) Show that the Riesz representation theorem does not hold for an incomplete inner product space.

UNIT III

VII. (a) Let H be a Hilbert space and A c BL (H). Show that there is a unique B c BL (H) such that for all x, y c H,

$$(A\ (x),\ y) = (x,\ B\ (y)).$$

(b) Let H be the Hilbert space K^2 and A : H —> H be defined by :

A $(x(1), x(2)) = (x(2), x(1))$ for $(x(1), x(2))$ s H. Show that A* = A.

VIII. (a) Let H be a Hilbert space and A s BL (H) -be self-adjoint. Show that A or – A is a positive operator if and only if

$$\langle A(x), y \rangle^2 \quad \langle A(x), x \rangle \text{ (A (y), y).}$$

(b) Let H be a non-zero Hilbert space and A c BL (H) be self adjoint. Show that :

$$\{m_A \text{ MA}\} \text{ C } \sigma_a \text{ (A)} = \text{(A)} \subset [m_A \text{ MA 1.}$$

IX. State and prove spectral theorem for compact self-adjoint operators.

(4 x 16 = 64 marks)