

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 4E 05—OPERATIONS RESEARCH

Time : Three Hours

Maximum : 36 Weightage

Part A*Answer **all** questions.**Each question has **weightage** 1.*

1. Briefly describe the minimum path problem.
2. For any feasible flow $\{x_i\}$, $i = 1, 2, \dots, m$ in the graph, prove that the flow x_0 in the return arc is not greater than the capacity of any cut in the graph.
3. State the problem of maximum potential difference in a network.
4. Describe the effect of deletion of Variables on the optimal solution of an LP problem.
5. What is goal programming.
6. Let $f(x)$ be a real-valued function in E_n , $G(X)$ a vector function consisting of real valued functions $g_i(x)$, $i = 1, 2, \dots, m$ as components and $F(X, Y) = f(x) + Y' G(X)$ where Y is a vector in E_m . If $F(X, Y)$ has a saddle point (X_0, Y_0) for every $Y \geq 0$, prove that X_0 is a minimum of $f(x)$ subject to the constraints $G(X) \leq 0$.
7. Write Kuhn-Tucker conditions for the problem :

$$\text{Minimize } 16(x_1 - 2)^2 + (4x_2 - 9)^2$$

$$\text{subject to } x_1 - x_2^2 \geq 0$$

$$x_1 + x_2 \leq 6,$$

$$x_1, x_2 \geq 0.$$

8. What is the advantage of solving the dual problem in geometric programming problems.
9. Write the general form of a geometric programming problem.
10. Define the decision variables and the state variables in a dynamic programming problem.
11. Explain the difference between forward recursion and backward **recursion** in **dynam** programming.

Turn over

12. Show that the function $f(x) = x^2$, $0 < x < 1$ is unimodal in $(0, 2)$.
13. What is meant by a sequential search plan.
14. Find the minimal point of $x^3 - 3x + 2$, $0 < x < 3$, taking $\epsilon = 0.01$ by the method of false position.

(14 x 1 = 14 weightage)

Part B

*Answer any seven questions.
Each question has weightage 2.*

15. Briefly describe an algorithm to find the spanning tree of minimum length if the length of each arc of the graph is given as a non-negative number.
16. A project consists of activities A, B, C, —M. In the following data $X - Y = C$ means Y can start after C days of Work on X. A, B, C can start simultaneously. K and M are the last activities and take 14 and 13 days respectively. $A - D = 4$, $B - F = 6$, $B - E = 3$, $C - E = 4$, $D - H = 5$, $D - F = 3$, $E - F = 10$, $F - G = 4$, $G - I = 12$, $H - I = 3$, $H - J = 3$, $J - K = 8$, $I - K = 7$, $I - L = 7$, $L - M = 9$. Find the least time of completion of the project.
17. For the problem :

$$\begin{aligned} \text{Maximize } f &= x_1 - x_2 + 2x_3 \\ \text{subject to } & -x_2 + x_3 \leq 4, \\ & x_1 + x_2 - x_3 \leq 3 \\ & 2x_1 - 2x_2 + 3x_3 \leq 15 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

assuming x_4, x_5, x_6 respectively as the slack variables for the three constraints, the optimal table is the following :

Basis	Values	x_1	x_2	x_3	x_4	x_5	x_6
x_3	21	4		1		2	1
x_4	7	2			1	1	0
x_2	24	5	1			3	1
$-f$	18	2				1	1

Carry out sensitivity analysis when the coefficient of x_1 in the objective function changes to 2.

8. Minimise $f = x_1^2 + x_2^2$
subject to $(x_1 - 1)^3 - x_2^2 > 0$.

9. Mention briefly an algorithm for solving a quadratic programming problem.
10. Explain the terms weight functions and normalised weight functions in geometric programming problems.

21. Describe a method of dynamic programming to solve the problem $z = \sum_{j=1}^n f_j(u_j)$

subject to $\sum_{j=1}^n a_j u_j$

$$a_j, b \in \mathbb{R}, a \geq 0, b > 0$$

22. Minimize $u_1 + u_2 + u_3$
subject to $u_1 + u_2 + u_3 = 10, u_1, u_2, u_3 \geq 0$

by forward recursion.

23. Briefly describe the computational algorithm for Fibonacci search plan.

24. Find the point of minimum of $f(x) = e^{-x} + x^2$ in the interval $(0, 1)$ using golden section method.
(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. Find the maximum non-negative flow in the network described below, arc (V_j, V_k) being denoted as (j, k) , V_s is the source and V_t is the sink.

Arc	(a, 1)	(a, 2)	(1, 2)	(1, 3)	(1, 4)	(2, 4)	(3, 2)	(3, 4)	(4, 3)	(3, t)	(4, t)
Capacity	8	10	3	4	2	8	3	4	2	10	9

26. Minimize $f(X) = (1 + X) x_1 + (-2 - 2x_1) x_2 + (1 + 5x_1) x_3$,
subject to $2x_1 - x_2 + 2x_3 \leq 2$,

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_1 + 2x_2 - 2x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

27. Minimise $x_2^2 + x_3 - 5x_1$
subject to $5x_1 - 3x_2 + x_3^2 \leq 2$

the variables being all positive.

28. Maximize $\sum_{n=1}^4 (4u_n - nu_n)$
subject to $\sum_{n=1}^4 u_n = 10, u_n \geq 0$.

(2 x 4 = 8 weightage)