C 42507 (Pages : 4) Name ....

Reg. No......................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2013

(CUCSS)

Mathematics

MT 4E 05—OPERATIONS RESEARCH

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Explain the term connected graph.
2. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
3. What is sensitivity analysis in an LP problem ?
4. Discuss the effect of introducing a new constraint on the Optimal Solution of an LP problem.
5. What do you mean by goal programming ?
6. Let $f(X)$ be a real valued function in E_n, $G(X)$ a vector function consisting of real valued functions $g_i(X), i = 1,2.....,m$, as components and $F(X, Y) = f(X) + y'G(X)$ where Y is a vector in E_n, . If F (X, Y) has a saddle point (X_0, Y_0) for every $y > 0$, prove that $G(X_0)$ 5_ 0.
7. Write the Kuhn-Tucker conditions for the problem

   Minimize **f =16** $_1 - 2)^2 + (4x_2 - 9)^2$

   subject to

   $-x_2^2 \quad 0$

   $x_1 + x_2 < 6$

   x_1, x_2 O.
8. Write the Kuhn-Tucker conditions for the quadratic programming problem.
9. Explain the term posynomial in Geometric programming.
10. Write the general form of geometric programming problem.
11. State Bellman's principle of optimality.

Turn over

12. Describe a method in dynamic programming to solve the problem

$$\text{Minimize } \sum_{j=1}^{n} f_j(u_j)$$

$$\text{subject to } \sum_{j=1}^{n} u_j > k > 0, u_i$$

13. Describe briefly the **Rosenbrock** method to locate the minimum of a function.

14. Explain **unimodality** in the case of a real-valued multi-variable function, $f(X)$, $X \in E_n$, $n \geq 2$.

(14 x 1 = 14 **weightage**)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Show that if $\{x_i\}$ and $\{y_i\}$ are two flows in a graph, then $\{ax_i + by_i\}$ where a and b are real constants is also a flow.

16. A building activity has been analyzed as follows, v_j stands for a job.

    (a) v_1 and v_2 can start simultaneously, each one taking 10 days to finish.

    (b) v_3 can start after 5 days and v_4 after 4 days of starting v_1.

    (c) v_4 can start after 3 days of work on v_3 and 6 days of work on v_2.

    (d) v_5 can start after v_1 is finished and v_2 is half done.

    (e) v_3, v_4 and v_4 take respectively 6, 8 and 12 days to finish.

    Draw the graph of the activity and find the critical path.

17. The following table gives the optimal solution to a LP problem of the type

    Maximize $f = CX$

    subject to $AX = B$, $X\ 0$.

Basis	*Values*	x_1	x_2	x_3	x_4	x_5
x_1	1	1		1	3	-1
x_2	2		1	1	-1	2
	8			4	3	4

x_4, x_5 are the slack variables respectively in the two constraints with right hand sides b_1 and b_2. The values of the cost coefficients are $C_1 = 2$, $C_2 = 3$, $C_3 = 1$. How much can the coefficient C_1 be increased before the current basis ceases to be optimal ?

18. Find the minimum of

$$f(X) = (x_1 + 1)^2 + (x_2 - 2)^2$$

subject to

$$g_1(X) = x_1 - 2 \le 0$$
$$g_2(X) = x_2 - 1 \quad O$$
$$x_1, x_2 \quad O.$$

19. Mention briefly the Wolfe's algorithm for solving a quadratic programming problem.

20. Show that Kuhn-Tucker conditions fail to give max x_1 subject to $(1- x_1)^3 - x_2$ O, x_1 O, x_2 O.

21. Using Kuhn-Tucker conditions verify that the solution $x_1 = 3.2$, $x_2 = 16^{-4} \times 10^4$, $x_3 = 4 \times 16^4 \times 10^2$, $x_4 = 10/(2 \times 16^2)$, $x_5 = 2$ minimizes $f_0 =$

subject to $f_1 = 2 \times 10^{-4} x_1 x_2\ 4\ x_5^{1} + 4 \times 10^4\ x_1 x_3^{-}\ x41\ x^{5^{1}} + 4x_1 x_4 x_5^{1} + x_5^{-1}$ 1 ,

$$f2 = X1\ x_2^{1/}\ x_5^{1} > 1,$$

$$x_1, x_2, \ldots, x_5 > 0 \bullet$$

22. Determine Max $(u_1 u_2 u_3)$

subject to $u_1 + u_2 + u_3 = 5$

$u_1, u_2, u_3 \geq 0.$

23. Solve :

Minimize $u_1^2 + u2 + u_3^2$

subject to

$$u_1 + u_2 + u_3 \ge 10,$$
$$u_1, u_2, u_3 \quad O$$

by forward recursion.

24. Write the computational algorithm of Fibonacci search method.

(7 x 2 = 14 weightage)

Turn over

Part C

Answer any **two** *questions.*
Each question has weightage 4.

25. Five villages in a hilly region are to be connected by roads. The direct distance (in km) between each pair of villages along a possible road and its cost of construction per km (in 10^4 rupees) are given in the following table (distances are given in the upper triangle and costs in the lower triangle). Find the minimum cost at which all the villages can be connected, and the roads which should be constructed.

		Distances				
		1	2	3	4	5
	1		18	12	15	10
	2	3		15	8	22
Costs	3	4	3		6	20
	4	5	5	6		7
	5	2	2	5	7	

26. Solve :

Minimize $f(X) = (1+\lambda)x_1 + (-2-2\lambda)x_2 + (1+5\lambda)x_3$

subject to

$2x_1 - x_2 + 2x_3 \;\; 2 + 2k,$

$x_1 - x_2 \le 3 + \lambda,$

$x_1 + 2x_2 - 2x_3 \;\; 4 - 4k,$

$x_1, x_2, x_3 \;\; 0.$

27. Solve :

Maximize Z $\quad -x_1 + x_2$

subject to

$2x_1 + 3x_2 < 6$

$2x_1 + x_2 \; 5 \; 4$

$x_1, x_2 \ge 0.$

28. Use the method of steepest ascent to go two steps towards the maximum of

$f(X) = \quad - x2 \; -4 \; -44$ starting at the point (-1, 1, 0, -1).

(2 x 4 = 8 weightage)