C 61267 **(Pages 4)** Name

Reg. No. . . . .

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2014

(CUCSS)

Mathematics

MT 4E 05—OPERATIONS RESEARCH

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. **Give an example to show that spanning tree of a graph need not be unique.**

2. **Write the problem of maximum flow in the generalized form.**

3. **Tasks A, B, C, . . H, I constitute a project. The notation X < Y means that the task X must be finished before Y can begin. With this notation,**

   A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I,

   draw a graph to represent the sequence of tasks.

4. **Describe the effect of introducing the constraint $3x_1 - 2x_2$ 2 in the L.P. problem**

   Minimize $Z = 4x_1 + 5x_2$

   subject to $2x_1 + x_2$ 6
   $x_1 + 2x$ 5_ 5
   $x_1 + x_2 \geq 1$
   $x_1 + 4x_2$ 2
   x_1, x_2 0.

   Whose optimal solutio 1 is $x_1 = 2/3$, $x_2 = 1/3$.

5. **What do you mean by parametric programming ?**

6. **Let $f(X)$ be a real-valued function in E_n, G (X) a vector function consisting of real-valued functions g ; (X), $i = 1, 2, \ldots, m$ as components and**

   $$F(X, Y) = f(X) + Y' G(X)$$

   where Y is a vector in E_m. If F (X, Y) has a saddle point (X_0, Y_0) for every Y 0, prove that X_0 is a minimum of $f(X)$ subject to the constraints G (X) 5_ 0.

Turn over

7. Write the Kuhn-Tucker conditions for the problem :

Minimize $f = x_1^2 + x_2^2$

subject to $x_1 + x_2$ 4

$2x_1 + x_2$ 5

8. What is the advantage of solving the dual problem in a geometric programming problem.

9. What is the difference between a posynomial and a polynomial.

10. Describe a method of dynamic programming to solve the problem

Maximize $j = 1$ $f_J (u_J$

subject to $\sum_{j=1}^{n} a_J u_J = k \quad u_J > 0,\ a_J > 0$

11. Define the term forward recursion as used in dynamic programming.

12. Solve by the method of dynamic programming

Maximize $\phi_2 = f_2\, f_1$ where $f_1 = u1,\ f_2 = u_2$

subject to $1 < u1 \leq 3,\ -1 < u2\ 1$.

13. Show that the function $f(x) = x^2,\ 0\ x < 1$ is unimodal in (0, 2).

14. Find the minimal point of $x^3 - 3x + 2$, 0 5 x 3 by Newton-Raphson method.

(14 x 1 = 14 weightage)

Part B

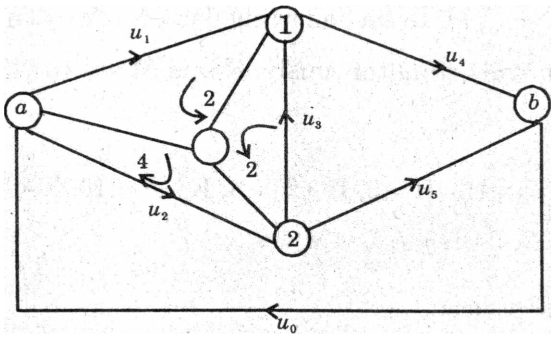
Answer any **seven** *questions.*
Each question has weightage 2.

15. Define the following terms :

(a) Tree ; (b) Spanning Tree.

16. Find the maximum flow in the following graph with the constraints

2 5_ x_i 10, 4 5_ x_2 12, −2 5_ x_3 5_4, 0 5_ x_4 5_5, 0 5_ x_5 5_10



17. **Describe the effect of introducing new variables on the optimal solution of an L.P. problem.**

18. **Solve graphically :**

Maximize $(x_1 - 4)^2 + (x_2 - 4)^2$

subject to $x_i + x_2$ 5_ 6

$x_i - x_2$ 1

$2x_1 + x_2$ 6

½ $x_1 - x_2$ − 4

0, x_2 0.

19. **State Kuhn-Tucker theorem.**

20. **Write the orthogonality conditions in a general geometric programming problem.**

21. **What are the essential features of dynamic programming problem.**

22. **Minimize : $u_1^2 + u_2^2 + u_3^3$**

subject to u_1 + u2 + 10

u_1 , U2 U3 a' O.

23. Briefly describe the Fibonacci search plan.

24. Find the minimal point of $x^3 - 3x + 2$, O $< x <$ 3 by quadratic interpolation.

(7 x 2 = 14 weightage)

Turn over

Part C

Answer any two questions.
Each question has weightage 4.

25. A project consists of activities A, B, C, . . , M. In the following data, X – Y = C means Y can start after C days of work on X. A, B, C can start simultaneously. K and M are the last activities and take 14 and 13 days repectively.

A – D = 4, B – F = 6, B – E = 3, C – E = 4, D – H = 5, D – F = 3, E – F = 10, F – G = 4, G – I = 12, H-I= 3,H-J= 3,J-K= 8,I-K= 7,L-M= 9.

Find the least time of completion of the project.

26. A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B is Rs. 40. The maximum demand of A is 6 units per week, and of B it is 8 units. The manufacturer has set up a goal of achieving a profit of Rs. 640 per week. Formulate the problem as goal programming, and sovle it.

27. Solve by the method of quadratic programming :

Minimize $-6x_1 + 2x_1^2 - 2x_1 x_2 + 2x_2^2$

subject to $x_1 + x_2 < 2$,

0, X2 O.

28. Find the maximum of f $(x) = -0.55 + 3x - x^2$ by Rosenbrock algorithm starting from $x = 0$, $h = 1$.

(2 x 4 = 8 weightage)