

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS-UG)

Mathematics—Core Course

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

Time : Three Hours

Maximum : 80 Marks

Part A

*Answer all the twelve questions.**Each question carries 1 mark.*

1. Evaluate : $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$
2. Define absolute convergence.
3. Find the focus of the parabola $y^2 = 10x$.
4. Evaluate $\int \coth 5x \, dx$
5. Find the Taylor polynomial of order 1 generated by $f(x) = \ln x$ at $a = 1$.
6. Write the parametric equations of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
7. Examine whether $\sum_{n=1}^{\infty} (-1)^{n+1}$ converges or diverges.
8. Examine whether $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents a parabola, ellipse or hyperbola.
9. Evaluate $\frac{d}{dx} \log_{10} (3x + 1)$.
10. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$
11. Show that x^2 grows faster than $\ln x$ as $x \rightarrow \infty$.
12. State **Leibniz's** theorem for an alternating series.

(12 x 1 = 12 marks)

Turn over

Part B

Answer any **nine** questions.
Each question carries 2 marks.

13. Graph the set of points whose polar co-ordinates satisfy the conditions $r < 0$ and $\theta = \frac{\pi}{4}$.

14. For what values of x do the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2^{n-1}} \text{ converges.}$$

15. Find k if $e^k = 10$.

16. Find the Maclaurin's series for $f(x) = \frac{1}{1+x}$

17. Find an equation for the hyperbola with eccentricity $\frac{3}{2}$ and directrix $x = 2$.

18. Evaluate $\int_0^{\pi/2} 2^{\sin x} \cos x \, dx$

19. Examine whether the series :

$$5 + \frac{2}{3} + 1 + \frac{1}{7} + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots + \frac{1}{k!} + \dots \text{ converges.}$$

20. Examine whether $x^2 + xy + y^2 - 1 = 0$ represents a parabola ellipse or hyperbola.

21. Prove that the alternating series :

$$\frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \dots \text{ converges.}$$

22. Examine whether $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges or diverges.

23. Prove that $e^{x+\ln 2} = 2e^x$

24. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

(9 x 2 = 18 marks)

Part C

Answer any **six** questions.
Each question carries 5 marks.

25. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{x+1} \right)$.

26. Find the tangent to the right-hand hyperbola branch $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $y = twit$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$ where $t =$.

27. Graph the curve $r = 1 - \cos \theta$.
28. Find the Maclaurin's series for $f(x) = \sin 3x$.
29. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!^{2n}}$.
30. Find the Taylor polynomial generated by :
 $I(x) = ex$ at $x = 0$.
31. Evaluate $\int \sinh^{-1} x \, dx$.
32. Find the polar equation for the circle $x^2 + (y - 3)^2 = 9$.
33. Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

(6 x 5 = 30 marks)

Part D

*Answer any two questions.
 Each question carries 10 marks.*

34. Solve the initial value problem :

$$e' \frac{dy}{dx} = 2x, x > \sqrt{3}, y(2) = 0$$

35. Find the length of the **cardioid** $r = 1 - \cos \theta$.

36. Using Integral test show that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ converges if $p > 1$ and diverges if $p \leq 1$.

(2 x 10 = 20 marks)