D 11545 **(Pages : 3)** **Name**

Reg. No..........................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Mathematics

MM 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 30 Weightage

I. Answer *all* questions :

1 Curvature of a straight line is ________

2 Find the parametric equation for the line through the points P$(-3,2,-3)$ and Q$(1,-1,4)$.

3 The domain of the function w = sin (x y) is the entire function. Then range = ________

4 If $\vec{r}(t) = (\cos t)i + (\sin t)j + tk$ then $\lim_{t \to \frac{\pi}{4}}$ (t) – ________

5 Vector equation for the line through $p_0(x_0\ y_0, z_0$ and parallel to $\vec{v}$ is $p_o p$ – ________

6 Find — if $f(x, y) = x^2 + 3xy + y - 1$ at (4,- 5).

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7 $(x, y) \to \left(0, \frac{\pi}{4}\right)$ $\sec x \tan x =$ ________

8 Find the gradient of $g(x, y) = y - x^2$ at **-1,0)**.

9 Define critical point.

10 Find the gradient field of $f(x,y,z) = xyz$.

11 **If F** is a field defined on D and F = V **[for** some scalar function f on D then f is called a ________ of F.

12 Examine whether $\mathbf{F} = y\,z\,i + z\,x\,j + x\,y\,k$ is conservative.

(12 x $^1/_4$ = 3 weightage)

Turn over

II. Answer any *nine* questions :

13 Show that $\vec{u}(t) = (\sin t)\, i + (\cos t)\, j + \quad k$ is orthogonal to its derivative.

14 Evaluate $\int_0 \left[t\, i + 7j + (t+1)k \right] dt$

15 Find the parametric equation for the line that is tangent to the curve :

$\vec{r}\,(t) = (a \sin t)\, i + (a \cos t)\, j + b\, t\, k, t_0 \quad 2\pi$.

16 Find the unit tangent vector of the curve :

$\vec{r}(t) = \left(\cos t\right) j + \left(\sin t\right) k,\ 0 \quad t \quad \frac{}{2}$.

17 Write the range of the function :

$f(x, y) = 4x^2 + 9y^2$.

18 If $w = xy + x\ y + x\ y$. Verify that $w_{xy} = w$.

19 If $w = x2 \quad y^2 + z^2$ and $z = x^2 + y2$, find $\left| \partial z \right|$.

20 Find ∇f at (1, 1, 1) of $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$

21 Evaluate $3 \int\int_{0\,0} (\quad y) \, dy \, dx$

22 Find the divergence of $\mathbf{F}(x, y) = (x^2 - y)\, i + \left(xy - y^2\right)$

23 State Green's theorem (Normal form).

24 State divergence theorem.

x 1 = 9 weightage)

III. Answer any *five* questions :

25. Find the distance from the point (2, 1, 3) to the line $x = 2 + 2t,\ y = 1 + 6t,\ z = 3$.

26 Find the plane through the points (1,1,-1), (2,0,2), (0,-2,1).

27 Prove that the curvature of a circle of radius a is $\frac{1}{a}$.

28 Find the Torsion for the helix :

$\vec{r}(t) = (a \cos t)\, i + a \sin t)\, j + b\, t\, k, \qquad 0,\ a^2 + b^2\ \mathbf{O}$.

29 Show that the function $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches (0, 0).

30 Find the linearization of $f(x,y,z) = x^2 + y^2 + z^2$ at (0, 1, 0).

31 A fluid's velocity field is $F = xi + zj + yk$. Find the flow along the helix :

$\vec{r}(t) = (\cos t)i + (\sin t)j + t\,k,\ 0 \le t \le 2.$

32 Evaluate $\iint e^{x^2+y^2}\,dy\,dx$ where R is the semi-circular region bounded by the x-axis and the curve $y = \sqrt{1-}$

(5 x 2 = 10 weightage)

IV. Answer *both the* questions :

33 Find the point of intersection of the lines $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$ and $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$ and then find the plane determined by these lines.

34 Find a potential function f for the field $F = 2xi + 3yj + 4zk$.

(2 x 4 = 8 weightage)