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(Pages : 3)

Name

Reg. No................................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Mathematics

MM 5B 06—ABSTRACT ALGEBRA

Time : Three Hours

Maximum : 30 Weightage

Part A

Questions from 1 to 12 *are* Compulsory.
Each has weightage ¼.

1. Give an example of a binary operation on the set of integers 'Z'.
2. State True *or* False :

   "$(R, +)$ is isomorphic to $(R^+, \cdot)$ " where '+' and '.' are the usual addition and multiplication respectively.
3. The number of non-trivial proper subgroup of Z_4 is ________
4. Find the number of generators of a cyclic group of order 5.
5. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ are two permutations in S_6. Compute $\sigma\tau$.
6. Define 'transposition' in S_n.
7. Find the left coset of the subgroup 3 Z of Z containing 1.
8. If ϕ is a homomorphism of a group G into a group G', then for $a \in G$ $\phi(a^{-1}) =$ ________
9. How many units are there in the ring of integers ?
10. Find the characteristic of Z_n.
11. What are the subspaces of the vector space R ?
12. State True *or* False :

    "Any subset S of a vector space V containing the zero vector is always linearly dependent".

(12 x ¼ = 3 weightage)

Turn over

Part B (Short Answer Type Questions)

Answer **all nine** *questions.*
Each question has weightage **1.**

13. Is the set-off all non-negative integers (including 0) under addition a group ? Give reasons.

14. Describe all the elements in the cyclic subgroup of GL (2, R) generated by $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

15. Prove that every cyclic group is Abelian.

16. Express the permutation $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{vmatrix}$ of S_8 as a product of disjoint cycles and then as a product of transpositions.

17. Find the orbits of the permutation a : z where $\sigma(n) = n + 2$.

18. State and prove Lagrange's theorem.

19. Define a ring and give example.

20. Solve the equation $x^2 - 5x + 6 = 0$ in Z_{12}.

21. Is the set of all polynomials of degree 'n' with usual rule of addition and multiplication a vector space ? Explain your answer.

(9 x 1 = 9 weightage)

Part C (Short Essay Questions)

Answer any **five** *questions.*
Each question has weightage 2.

22. If G is a group with binary operation x, prove that the left and right cancellation laws hold in G.

23. Show that the collection of all permutation of the set {1, 2, 3) is a group under permutation multiplication.

24. Prove that a subgroup of a cyclic group is cyclic.

25. Let ϕ be a homomorphism of a group G into a group G and H a subgroup of G, then prove that $[H]$ is a subgroup of G'.

26. If R is a ring with additive identity '0', then for any $a, b \in R$. Prove the following :

    (a) $a(-b) = (-a)b = -(a,b)$.

    (b) $(-a)(-b) = ab$.

27. If P is a prime, then prove that Z_p is a field.

28. Find 'k' such that 42,-1,3), (3,4,-1), (k,2,1)} is linearly independent.

(5 x 2 = 10 weightage)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weightage 4.

29. (a) State and prove a necessary and sufficient condition for a non-empty subset H of a group G to be a sub-group of G.

    (b) Show that if H and K are subgroups of abelian group G, then $\{hK \; h \in H, k \in K\}$ is also a subgroup of G.

30. (a) Define even permutations and give example.

    (b) If n 2, then prove that the collection of all even permutations of {1,2,3, , n} forms a subgroup of order $^{-!}/_{2}$ of the symmetric group S_n.

31, (a) Define dimension of a vector space.

    (b) Find the dimension of the subspace $U = \{(x_1, x_2, x_3) / x_1 - x_2 + x_3 = 0\}$ of $\mathbf{R}^3$ by finding a basic for U.

    (c) Let U and W be two subspaces of a finite dimensional vector space V. Then prove that :

    dim (U + W) dim U + dim W -- dim (U $\cap$ W).

(2 x 4 = 8 weightage)