D 11547 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER B.Sc. DEGREE (SUPPLEMENTARY/IMPROVEMENT) EXAMINATION, NOVEMBER 2016

(UG–CCSS)

Mathematics

MM 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 30 Weightage

I. Objective type questions. Answer *all twelve* questions :

1 ~~Let~~ $\cancel{f(x)=\frac{2x}{x-1}}$ ~~, for all~~ $\cancel{x \in A=\{x \in R : x \neq 1\}}$ ~~, then range of~~ ~~*f*~~ ~~is~~

2 Using algebraic properties of R, prove that $a+b=0 \Rightarrow b=-a$.

3 Find the supremum of $S=\left\{1\ \frac{(-1)^n}{n} : n\ E\ N\right\}$.

4 State nested intervals property.

5 Give an example of a convergent sequence of positive numbers with $\lim x_n^{1/n}=1$.

6 Show that $\{2\ \}$ cannot converge.

7 Define a Cauchy sequence.

8 State True *or* False :

"If $\{x_n\}$ converges to 1, then every subsequence of $\{x_n\}$ also converges to *1"*.

9 Give an example of an open set in R.

10 State True *or* False :

"Arbitrary union of closed sets in R is closed".

11 Prove that $\mathrm{Im}(iz)=\mathrm{Re}\,z$, 'where z is a complex number.

12 Prove that 1 + 2's is closer to the origin then 3 + 4 i.

(12 x ¼ = 3 weightage)

Turn over

II. Very short answer questions. Answer *all nine* questions :

13 Suppose S and T are sets such that T c S then prove that If S is finite then T is finite.

14 Given $f = x^2 + 1$ and g find fog and g

15 If x and y are reals with x < y, then prove that if an irrational number of such that $x < z < y$.

16 Give examples to show that supremum and infimum may not belong to the set.

17 Find the supremum and Infimum of the set S where $S = \{x : 3x^2 - 10x + 3 < 0\}$.

18 Using the definition of limit, show that $\lim \left(\frac{3x+}{x+1}\right) =$

19 If $X = (x_n)$ is a convergent sequence of reals and if x_n 0 for all n, then prove that :

$x = \lim(x_n) \geq 0$.

20 Establish the proper divergence of $(\sqrt{n})$.

21 Find the least positive integer (non-zero) in such that $\left(\frac{1+i}{1-i}\right) = 1$.

(9 x 1 = 9 weightage)

III. Short answer questions. Answer any *five* questions :

22 Prove that there does not exist a rational number r such that $r^2 = 2$.

23 If a, b E R, prove that :

I b I | ≤ | a b |.

24 Let S be a non-empty bounded set in R and a > 0 and $aS = (as ; s$ E S} , prove that :

sup (aS) = a • sup S

25 Prove that a sequence of real numbers is convergent iff it is a Cauchy sequence.

26 Prove that the intersection of arbitrary collection of closed sets is closed.

27 Prove that I $z_1 + z2$ I ≤ I $z1$ I + | $z2$ I •

28 Locate the points in the complex plane for which I $z - 1$ I^2 + | $z + 1$ | $= 4$.

(5 x 2 = 10 weightage)

IV. Essay questions. Answer any *two* questions :

29. (a) Show that the set of all real numbers between 0 and 1 is uncountable.

    (b) If $x_n = \overline{} \; -$ show that $[x_n]$ converges.

30. Show that a monotonic sequence of real numbers is convergent iff it is bounded.

31. (a) Prove that $\arg(z_1 z_2) = \arg \quad + \arg z_2$.

    (b) Find the value of $2i$.

(2 x 4 = 8 weightage)